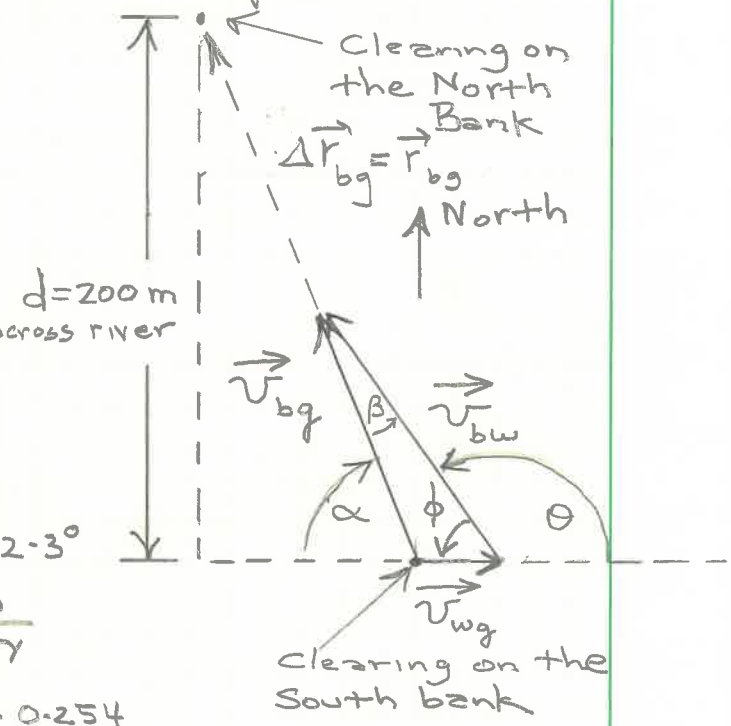
STAEDTLER® No. 937 811E Engineer's Computation Pad

82 A 200-m-wide river has a uniform flow speed of 1.1 m/s through a jungle and toward the east. An explorer wishes to leave a small clearing on the south bank and cross the river in a powerboat that moves at a constant speed of 4.0 m/s with respect to the water. There is a clearing on the north bank 82 m upstream from a point directly opposite the clearing on the south bank. (a) In what direction must the boat be pointed in order to travel in a straight line and land in the clearing on the north bank? (b) How long will the boat take to cross the river and land in the clearing?

Draw a Relative Velocity Addition Diagram:

$\vec{v}_{bg} = \vec{v}_{bw} + \vec{v}_{wg}$

where: b=boat; w=water g=ground



Navigational Nomenclature:

Bearing is the direction from the ship to the Destination

Heading is the Direction the ship is pointing as it is possibly moving through some moving medium like water or air.

$\tan\alpha = \frac{d}{x} \Rightarrow \alpha = \tan^{-1}\left(\frac{d}{x} = \frac{200}{82}\right)$

$\Rightarrow \alpha = 67.7°$

$\Rightarrow$ The Angle opposite side v_{bw} is $\gamma = 180° - \alpha = 180° - 67.7° = 112.3°$

The Law of Sines $\Rightarrow \frac{v_{wg}}{\sin\beta} = \frac{v_{bw}}{\sin\gamma}$

$\Rightarrow \sin\beta = \frac{v_{wg}}{v_{bw}}\sin\gamma = \frac{1.1}{4.0}\sin 112.3° = 0.254$

$\Rightarrow \beta = 14.74° \Rightarrow \phi = 180° - \beta - \gamma = 53.0$

a) $\therefore$ The Heading is $\theta = 180° - \phi = 127.0°$

The Law of Cosines $\Rightarrow v_{bg}^2 = v_{bw}^2 + v_{wg}^2 - 2v_{bw}v_{wg}\cos\phi$

$v_{bg}^2 = 4.0^2 + 1.1^2 - 2(4.0)(1.1)\cos 53 = 11.91 \Rightarrow v_{bg} = 3.45\ m/s$

The Displacement of the boat with respect to the ground $= |\Delta\vec{r}_{bg}| = |\vec{r}_{bg}| = \sqrt{d^2 + x^2} = \sqrt{200^2 + 82^2} = 216.2\ m$

If Δt is the time to cross the river

b) Then $r_{bg} = v_{bg}\Delta t \Rightarrow \Delta t = \frac{r_{bg}}{v_{bg}} = \frac{216\ m}{3.45\ m/s} = 62.6\ sec$

Using Components:

$\vec{v}_{bg} = \vec{v}_{bw} + \vec{v}_{wg} \Rightarrow +v_{bg}\cos\gamma\,\hat{x} + v_{bg}\sin\gamma\,\hat{y} = +v_{bw}\cos\theta\,\hat{x} + v_{bw}\sin\theta\,\hat{y} + v_{wg}\hat{x}$

or

$-v_{bg}\cos\alpha\,\hat{x} + v_{bg}\sin\alpha\,\hat{y} = -v_{bw}\cos\phi\,\hat{x} + v_{bw}\sin\phi\,\hat{y} + v_{wg}\hat{x}$

$\Rightarrow$ $-v_{bg}\cos\alpha = -v_{bw}\cos\phi + v_{wg}$ & $v_{bg}\sin\alpha = v_{bw}\sin\phi$ } 2 Eq^{n}s in 2 Unknowns ϕ & v_{bg} which can be solved