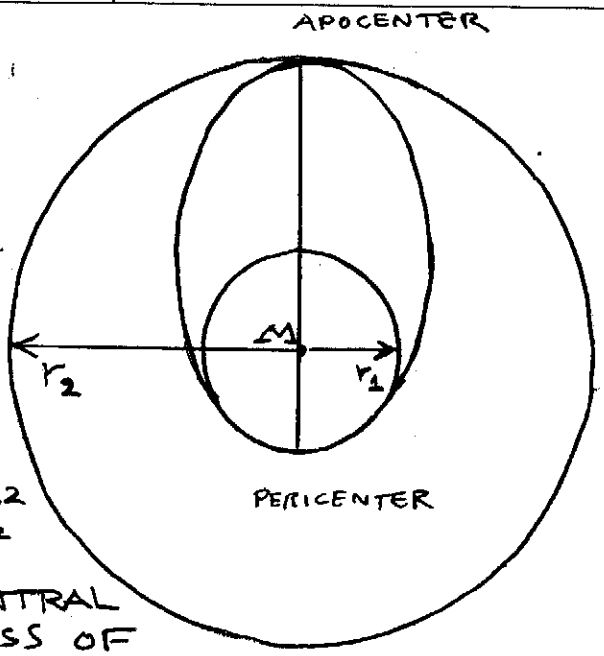
STAEDTLER® No. 937 811E Engineer's Computation Pad

**Hohmann Transfer Orbits:**
Used to transfer a spacecraft from:
one Circular Orbit to another.
Examples: Near-Earth Orbit to Lunar Orbit
Earth Orbit to Mars Orbit.
**Question:** Find the time necessary to travel to Mars or the Moon via this Elliptical Orbit




BY KEPLER'S 3RD LAW:
LAW OF PERIODS

$$a_1^3 \underset{\text{CIRCULAR}}{=} r_1^3 = \frac{G(m+M)}{4\pi^2} T_1^2 \approx \frac{GM}{4\pi^2} T_1^2$$

$$a_2^3 \underset{\text{CIRCULAR}}{=} r_2^3 = \frac{G(m+M)}{4\pi^2} T_2^2 \approx \frac{GM}{4\pi^2} T_2^2$$

HERE $M$ IS THE MASS OF THE CENTRAL MASSIVE BODY & $m$ IS THE MASS OF THE ORBITING BODY NOTE $M \gg m$

THE MAJOR AXIS, LENGTH $2a$, OF AN ELLIPSE IS THE LENGTH OF THE LONGEST DIAMETER - THE LINE OF APSIDES - WHICH CONNECTS THE 2 APSIDES: THE LOWER APSIS OR PERICENTER (SPECIFICALLY: ABOUT EARTH = PERIGEE & ABOUT SUN = PERIHELION) & THE UPPER APSIS OR APOCENTER (APOGEE & APHELION)

$r_1$ = 1ST CIRCULAR ORBIT RADIUS = $r_{min}$ = PERICENTER DISTANCE FOR THE Hohmann ELLIPTICAL ORBIT = $a_H(1-e_H)$ · $a$ = SEMIMAJOR AXIS OF ELLIPSE · $e$ = ECCENTRICITY OF ELLIPSE

$r_2$ = 2ND CIRCULAR ORBIT RADIUS = $r_{max}$ = APOCENTER DISTANCE FOR THE Hohmann ELLIPTICAL ORBIT = $a_H(1+e_H)$ : $a_H$ & $e_H$ ARE HOHMANN'S ORBIT

NOTE: $a_H = \frac{r_{min}+r_{max}}{2}$ AND IN THIS AVERAGING SENCE IS THE AVERAGE DISTANCE FROM $M$ IN ELLIPTICAL ORBIT.

KEPLER'S 3RD LAW FOR HOHMANN ORBIT: $a_H^3 = \left(\frac{r_1+r_2}{2}\right)^3 = \frac{GM}{4\pi^2} T_h^2$

EARTH'S TO MAR'S ORBIT: $r_1 = r_{Earth}$ & $r_2 = r_{MARS} = 1.5\, r_{EARTH} = 3/2\, r_E$

$$\Rightarrow T_H^2 = \frac{4\pi^2}{GM_{sun}} \underbrace{\left(\frac{5}{4} r_{EARTH}\right)}_{a_H}^3 = \frac{125}{64}\frac{4\pi^2}{GM} r_E^3 = \frac{125}{64}\left(T_E = 1\text{ YEAR}\right)^2 = 1.95\ \text{YEARS}^2$$

$\Rightarrow T_H = 1.40$ YEARS; BUT YOU ONLY NEED $T_H/2 = 0.7$ YEARS TO GET TO MARS

LEO (LOW-EARTH ORBIT) TO LUNAR ORBIT: $r_2 = r_{moon}$ & $r_1$ = RADIUS OF EARTH $= \frac{1}{60} r_{moon}$

$$\Rightarrow T_H^2 = \frac{4\pi^2}{GM_{EARTH}} \underbrace{\left[\frac{1}{2}\left(1+\frac{1}{60}\right) r_{moon}\right]}_{a_H}^3 = \left(\frac{61}{120}\right)^3 \frac{4\pi^2}{GM_E} r_{moon}^3 \approx \left(\frac{1}{2}\right)^3 \left(T_{moon} = 27.3\text{ days}\right)^2$$

$\Rightarrow T_H = 9.65$ DAYS (9.89 if I USED $\left(\frac{61}{120}\right)^3$ INSTEAD OF $\frac{1}{8}$) $\Rightarrow T_H/2 \approx 5$ DAYS TRAVEL TIME TO MOON

**Question:** Use Energy Considerations to Determine how much Energy is needed (and resulting velocity boost (ie $\Delta v$)) to Transfer from the 1st Circular Orbit to the Elliptical Hohmann Orbit, and then to the 2nd Circular Orbit.

$$E^{\text{total}}_{\text{mechanical}} = K + U = \frac{1}{2} m v^2 + \frac{-GmM}{r} = \frac{-GmM}{2a} = \text{constant for any orbit with } a = \text{its semi-major axis}$$

$E^{\text{total}}_{\text{orbit \#1}} = -GmM/2r_1$ add Energy to get

$E^{\text{total}}_{\text{Hohmann}} = -GmM/2a_H$ add Energy to get

$E^{\text{total}}_{\text{orbit \#2}} = -GmM/2r_2$ Hohman is the TRANFER ORBIT

CONTINUED...

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... COMPARE THE TOTAL ENERGIES FOR SPACECRAFT, m.

$-\frac{GmM}{2r_1}$ FOR |1⟩ EARTH'S ORBIT ABOUT SUN
|2⟩ LOW EARTH ORBIT

$-\frac{GmM}{2a}$ FOR |1⟩ HOHMANN TRANSFER ORBIT TO MARS
|2⟩ HOHMANN TRANSFER ORBIT TO MOON

$-\frac{GmM}{2r_2}$ FOR |1⟩ MAR'S ORBIT ABOUT SUN
|2⟩ LUNAR ORBIT ABOUT EARTH

USE: $m = 10$ METRIC TONS $= 10{,}000$ kg
$M$ = EARTH'S MASS $= 5.98 \times 10^{24}$ kg
$M$ = SUN'S MASS $= 1.99 \times 10^{30}$ kg
$r_1$ = RADIUS OF EARTH $= 6.37 \times 10^6$ m
$r_1$ = RADIUS OF EARTH'S ORBIT $= 1.5 \times 10^{11}$ m
$r_2$ = RADIUS OF LUNAR ORBT $\simeq 60\, R_{EARTH} = 3.82 \times 10^8$ m
$r_2$ = RADIUS OF MARS ORBIT $\simeq 1.5$ EARTH'S $= 2.25 \times 10^{11}$ m
$a = \frac{r_1 + r_2}{2}$ : TO MARS $= 1.875 \times 10^{11}$ m
: TO MOON $= 1.94 \times 10^8$ m

$E^{total} = \frac{1}{2}mv^2 - \frac{GmM}{r} = -\frac{GmM}{2a} \Rightarrow v = \sqrt{GM\left(\frac{2}{r} - \frac{1}{a}\right)}$

$\Rightarrow v$ @ $r_1$ in original orbit: $v_{1,0} = \sqrt{GM\left(\frac{2}{r_1} - \frac{1}{r_1}\right)} = \sqrt{GM\frac{1}{r_1}}$

$v$ @ $r_1$ in Hohman orbit: $v_{1,H} = \sqrt{GM\left(\frac{2}{r_1} - \frac{2}{r_1 + r_2}\right)} = \sqrt{GM\frac{2r_2}{r_1(r_1+r_2)}}$

$v$ @ $r_2$ in Hohman orbit: $v_{2,H} = \sqrt{GM\left(\frac{2}{r_2} - \frac{2}{r_1 + r_2}\right)} = \sqrt{GM\frac{2r_1}{r_2(r_1+r_2)}}$

$v$ @ $r_2$ in LAST orbit: $v_{2,L} = \sqrt{GM\left(\frac{2}{r_2} - \frac{1}{r_2}\right)} = \sqrt{GM\frac{1}{r_2}}$

| ORBT | EARTH → MARS | | | EARTH → MOON | | |
|---|---|---|---|---|---|---|
| | ORBIT ENERGY | $v_{1,0}$ & $v_{1,H}$ | $v_{2,H}$, $v_{2L}$ | ORBIT ENERGY | $v_{1,0}$ & $v_{1,H}$ | $v_{2H}$ & $v_{2L}$ |
| ORIGINAL ORBIT | $-4.43\times10^{12}$ J + $+0.89\times10^{12}$ J = | 29,800 m/s ↓ | | $-313\times10^9$ J + $+302.7\times10^9$ J = | 7,920 m/s ↓ | |
| HOHMANN ORBIT | $-3.54\times10^{12}$ J + $+0.59\times10^{12}$ J = | 32,600 m/s → | 21,730 m/s ↓ | $-10.3\times10^9$ J + $+5.08\times10^9$ J = | 11,100 m/s → | 185 m/s ↓ |
| LAST ORBIT | $-2.95\times10^{12}$ J | | 24,300 m/s | $-5.22\times10^9$ J | | 1,020 m/s |