**57** An *astronomical unit* (AU) is equal to the average distance from Earth to the Sun, about $92.9 \times 10^6$ mi. A *parsec* (pc) is the distance at which a length of 1 AU would subtend an angle of exactly 1 second of arc (Fig. 1-8). A *light-year* (ly) is the distance that light, traveling through a vacuum with a speed of 186 000 mi/s, would cover in 1.0 year. Express the Earth–Sun distance in (a) parsecs and (b) light-years. SSM

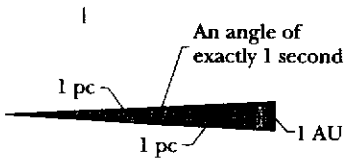
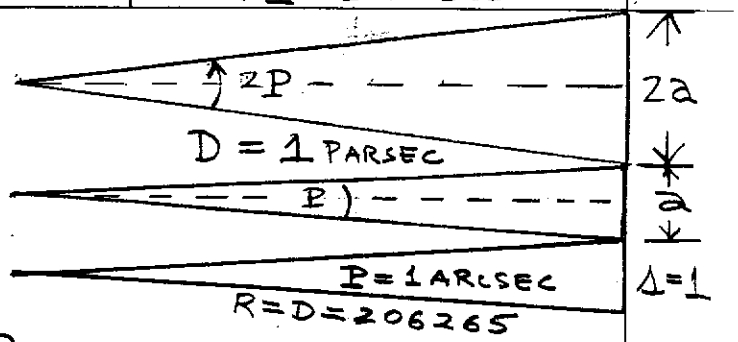



**FIG. 1-8** Problem 57.

HERE $a = 1\,AU = 149.6 \times 10^6$ km & $P = 1$ Arcsec




Our Book's Def$^n$ $\Rightarrow \sin P/2 = a/2/D$

& Astronomy Book Def$^n$ $\sin P = a/D$

agree since $P$ = parallax Angle is small and in Radian measure $\sin(\alpha) \approx \alpha$ if $\alpha$ is small & in Radians, since $\alpha \equiv \Delta/R$ = (Arclength/Radius) in Radians.

$$\text{ANGLE} = P \equiv 1\,\text{ARCSEC} \equiv \frac{1}{60}\,\text{ARMIN} = \frac{1}{60}\left[\frac{1}{60}\,\text{DEGREE}\right]$$

$$\overset{\text{in Rads}}{=} \frac{1}{3600}\,\text{DEGREE}\left[\frac{2\pi\ \text{RADIANS}}{360\ \text{DEGREES}}\right] = \left(\frac{1}{206265}\right)\text{RAD} = 4.84814 \times 10^{-6}\ \text{RAD}$$

To be precise $\sin P = \frac{a}{D} \Rightarrow D = \frac{a}{\sin P} = \frac{1\,AU}{4.84814 \times 10^{-6}}$

PARSEC $D = 206{,}265\,AU \approx 2.06 \times 10^5\,AU\left(92.9 \times 10^6 \frac{\text{miles}}{AU}\right)$

$= 19.2 \times 10^{12}$ miles

LIGHT YEAR: $d = vt = 186{,}000 \frac{\text{miles}}{\text{sec}}\left[1\,\text{YEAR}\left(\frac{365.2422\ \text{days}}{\text{YEAR}}\right)\cdots\right.$

$$\left.\cdots\left(\frac{24\,\text{hrs}}{\text{day}}\right)\left(\frac{60\,\text{min}}{\text{hr}}\right)\left(\frac{60\,\text{sec}}{\text{min}}\right)\right] = 5.87 \times 10^{12}\ \text{miles}$$

NOTE: $\frac{1\ \text{PARSEC}}{1\ \text{LIGHT YEAR}} = \frac{19.2}{5.87}\ \frac{\times 10^{12}\ \text{mi.}}{\times 10^{12}\ \text{mi.}} = 3.26 \Rightarrow \text{PARSEC} = 3.26\,\text{LY}$

$$1\,AU = 1\,AU\left(\frac{1\ \text{PARSEC}}{206{,}265\,AU}\right) = 4.84814 \times 10^{-6}\ \text{PARSECS}$$

$$1\,AU = 92.9 \times 10^6\ \text{miles}\ \frac{\text{LIGHT YEAR}}{5.87 \times 10^{12}\ \text{miles}} = 1.58 \times 10^{-5}\ \text{LYs}$$

CHECK $\frac{1.58 \times 10^{-5}\ \text{LY/AU}}{4.85 \times 10^{-6}\ \text{PS/AU}} = 3.26\ \frac{\text{LY}}{\text{PS}}$ ✓

ASIDE: CONVERSION FROM RADIANS TO ARCSEC: $206{,}265\ \frac{\text{ARCSEC}}{\text{RADIAN}}$

$\Rightarrow \alpha\,(\text{in Radians}) = \alpha\,(\text{in ARCSEC})/(206{,}265\ \text{ARCSEC/RADIAN})$

THE DISTANCE TO A STAR, D, IS RELATED TO ITS PARALLAX ANGLE, P, AND THE AU = a via $\sin P = a/D$

$\Rightarrow D = a/\sin P = a/P$ (in Radians since P is small)

$= \frac{a\,206{,}265}{P\,(\text{in ARCSEC})}$ [NOTE: IF a is in AU THEN D is in AU] $\Rightarrow \frac{D(\text{in AU})}{206{,}265} = \frac{1}{P\,(\text{in ARCSEC})} \Rightarrow \boxed{D = \frac{1}{P}}$

(in PARSEC) (in Arcsec)