STAEDTLER® No. 937 811E Engineer's Computation Pad

••21 The position of a particle moving along the x axis depends on the time according to the equation $x = ct^2 - bt^3$, where x is in meters and t in seconds. What are the units of (a) constant c and (b) constant b? Let their numerical values be 3.0 and 2.0, respectively. (c) At what time does the particle reach its maximum positive x position? From $t = 0.0$ s to $t = 4.0$ s, (d) what distance does the particle move and (e) what is its displacement? Find its velocity at times (f) 1.0 s, (g) 2.0 s, (h) 3.0 s, and (i) 4.0 s. Find its acceleration at times (j) 1.0 s, (k) 2.0 s, (l) 3.0 s, and (m) 4.0 s.

$$x(t) = ct^2 - bt^3$$

$$v(t) = \frac{dx}{dt} = 2ct - 3bt^2$$

$$a(t) = \frac{dv}{dt} = 2c - 6bt$$

a) $x(t) = ct^2 - bt^3$ must be dimensionally consistent, ie.

length = Length − Length $\Rightarrow ct^2$ units ~ meters

$\Rightarrow c$ units ~ $\frac{\text{meters}}{t^2} = \frac{m}{s^2}$

b) bt^3 units ~ Length $\Rightarrow b$ units ~ $\frac{\text{meters}}{\text{seconds}^3}$

c) $v = \frac{dx}{dt} \overset{\text{When}}{=} 0$ when $2ct - 3bt^2 = 0$ ($v=0$ when the particle reverses direction.)

x is a minimum or a maximum

or $t(2c - 3bt) = 0$

@ $t = 0$ & $2c - 3bt = 0 \Rightarrow t = 2c/3b = 2(3)/3(2) = 1$ sec

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = 2(3) - 6(2)t = 6 - 12t = 6(1 - 2t)$$

$$v = \frac{dx}{dt} = 2(3)t - 3(2)t^2 = 6.0t - 6.0t^2$$

$$= 6.0t(1 - t)$$

$$x = 3.0t^2 - 2.0t^3 = t^2(3.0 - 2.0t)$$

NOTES: $v(t=0) = 0$ & $v > 0$ for $t < 1$ s, then $v = 0$ @ $t = 1$ s

then $v < 0$ for $t > 1$ s

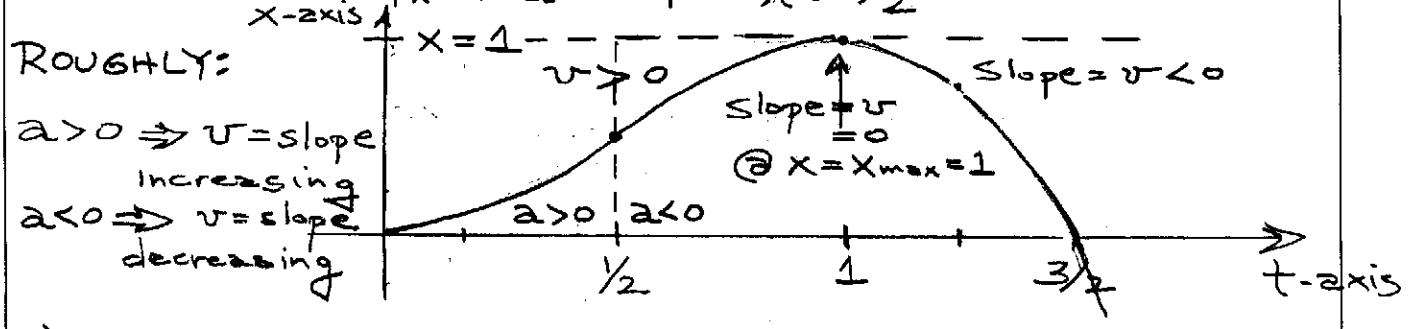
$x(t=0) = 0$ & $x > 0$ for $t < 3/2$ s, then $x = 0$ @ $t = 3/2$ s

then $x < 0$ for $t > 3/2$ s

$a(t=0) = 6$ which decreases to $a(t = \frac{1}{2}) = 0$

then $a < 0$ for $t > 1/2$

ROUGHLY:



$a > 0 \Rightarrow v$ = slope increasing

$a < 0 \Rightarrow v$ = slope decreasing

d) for $t = 0 \rightarrow 1$ sec, $x = 0 \rightarrow 1$ m, its maximum x value

$x(t=4) = 3.0(4)^2 - 2.0(4)^3 = -80$ m

for $t = 1 \rightarrow 4$ sec $x = 1 \rightarrow -80$ m

$\Rightarrow$ distance traveled $= 1 + 81 = 82$ m

TOTAL DISTANCE, NOT DISPLACEMENT

NOTE AVERAGE SPEED $= \frac{|82\text{m}|}{4\text{s}} = 20.5 \frac{m}{s}$

e) DISPLACEMENT $\equiv \Delta x = x_f - x_i = x(t=4) - x(t=0)$

$= -80 - (+1.0) = -81\ m$

NOTE AVERAGE VELOCITY $\equiv \frac{\Delta x}{\Delta t}$

$= \frac{-81\ m}{4\ s}$ $\left(\neq \frac{+82\ m}{4\ s}\right)$

$v(t) = 6.0\,t\,(1-t)$

f) $v(t=1.0) = 6(1)(0) = 0\ m/s$

g) $v(t=2.0) = 6(2)(-1) = -12.\ m/s$

h) $v(t=3.0) = 6(3)(-2) = -36\ m/s$

i) $v(t=4.0) = 6(4)(-3) = -72\ m/s$

$a(t) = 6.0\,(1-2.0\,t)$

j) $a(t=1.0) = 6(-1) = -6.0\ m/s^2$

k) $a(t=2.0) = 6(-3) = -18\ m/s^2$

l) $a(t=3.0) = 6(-5) = -30.\ m/s^2$

m) $a(t=4.0) = 6(-7) = -42.\ m/s^2$

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