The Law of Hydrostatics

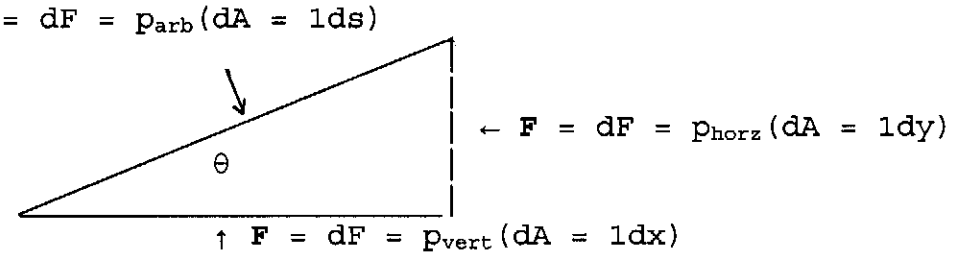
...from Richard Feynman in ...
The Flow of Dry Water - Feynman Lectures on Physics

"We begin by considering hydrostatics, the theory of liquids at rest. When liquids are at rest, there are no shear forces (even for viscous liquids). The law of hydrostatics, therefore, is that the stresses are always normal to any surface inside the fluid. The normal force per unit area is called the pressure. From the fact that there is no shear in a static fluid it follows that the pressure stress is the same in all directions. We will let you entertain yourself by proving that if there is no shear on any plane in a fluid, the pressure must be the same in any direction."

No Shear Forces => The Stresses are always Normal or Perpendicular to any Surface (Real or Imaginary) inside the Liquid.
ie The Net Force exerted by the Liquid to one side of a Unit Area on the Liquid to the other side of the Unit Area is Perpendicular to that Unit Area.

A Force can only be exerted on something that is capable of exerting an Equal & Opposite Force back.
For Example, consider a Heavyweight Boxer who can exert a Huge Force with his fist on a Full-sized Punching Bag. However, ... he can only exert a Tiny Force on a Tissue Paper floating before him.
Experiment in the BathTub with your Hand under Water: An open palm can exert a Force on the water (ie Push on the Water) perpendicular to its Surface, However, an oblique Karate Chopping action with the Palm exerts No Shearing Force on the Water it slices through.
A Rectangular Volume of Fluid will deform Continuously with no Resistance to a Shearing Force - no matter how small it is, while a Solid will resist deformation and Elastically reform from its distortion when the Shearing Force is discontinued.

After accepting that the Liquid can not support a Shear Force ...
Prove that the Pressure is the same in All Directions:
Consider a Right-Triangular Slab of Liquid within the entire Volume of a Liquid of Unit Thickness with the Vertical y-axis & the Horizontal x-axis aligned with the Right Angle:



Equilibrium for the Triangular Slab of Liquid => $\mathbf{F}_{net} \equiv 0$
in the x-direction: $+p_{arb}ds[\sin(\theta)] - p_{horz}dy = 0 \Rightarrow p_{arb} = p_{horz}$
in the y-direction: $-p_{arb}ds[\cos(\theta)] + p_{vert}dx - \frac{1}{2}dxdy(1)\rho g = 0 \Rightarrow p_{arb} = p_{vert}$
NOTE: $\sin(\theta)ds = dy$ & $\cos(\theta)ds = dx$ & the Volume of the Triangular Slab $= \frac{1}{2}dxdy(1)$ is the product of two Infinitesimals and therefore is of 2nd order of smallness which is ignorable compared to Calculus's first order of smallness; ie: $dxdy = 0$ for infinitesimal dx & dy

Therefore: $p_{arb} = p_{horz} = p_{vert}$