Calculus-based Physics 201

Test #3:

Center of Mass & Linear Momentum; Rotation, Angular Momentum, Torque; & Equilibrium

Fall 2025

Numerical Answers: Use the MKS Metric System unless explicitly told to do otherwise. Give Numerical Answers to at least Three Significant Figures.

NO: Notes, Formula Cards, Formulas or Constants in your Calculators, or any other aids. Vectors are indicated in Bold Print (**v**).

Unit Vectors in the x, y, & z directions are symbolized by **i**, **j**, & **k** respectively. Multiple Choice are 5 Points each. Problems #1 & #2 are 15 & 20 Points.

One of the 13 Multiple Choice is a Bonus unless it is found that one of the Multiple Choice Questions is bad for some reason and must be eliminated.

Chapter 9, Elastic Collision in 1-dim -> $(v_{1i} - v_{2i}) = -(v_{1f} - v_{2f})$:

| 1> In a UNCW vs NCSU point particle competition a Teal Particle is traveling in the positive x-direction at a speed of 3.33 m/s while a Red Particle is traveling at 1.11 m/s in the negative x-direction. After they collide the velocity of the Teal Particle relative to the Red Particle is _____ m/s.

->a) -4.44 b) +4.44 c) -2.22 e) +2.22

e) Definitely need to know the masses to answer this Question!

Relative Velocity: Velocity of #1 relative to #2 = $v_{12} = v_{1,L} + v_{L,2} = v_{1,L} - v_{2,L}$

Relative velocity of approach = relative velocity of recession:

$(v_{1i} - v_{2i}) = -(v_{1f} - v_{2f})$ => v_{12} final = $(v_{1f} - v_{2f}) = -(v_{1i} - v_{2i}) = -(+3.33 - -1.11) =$ final $v_{12} = -4.44$

Chapter 9, Fext=0 => Center of Mass:

| 2> A tree-climbing dog of mass M=13.00kg and a cat of mass m=3.25kg get into a fight in a tree. The animals become entangled and fall vertically downward together from a limb. They fall off the limb at a distance of 2.0 m from the straight trunk of the tree. During the fall the animals separate. If the dog lands at the base of the tree's trunk, then how far from the tree's trunk in meters does the cat land?

a) 4 b) 6 c) 8 ->d) 10 e) 34

$\mathbf{r}_{com} = (m_1\mathbf{r}_1 + m_2\mathbf{r}_2)/(m_1 + m_2)$ => Md = mD => D = Md/m = 13(2)/3.25 = 8 => 8+2=10

or $\mathbf{r}_{com} = 2 = (m_1\mathbf{r}_1 + m_2\mathbf{r}_2)/(m_1 + m_2) = (M(0) + mx)/(m+M)$ => x = 2(m+M)/m

= 2(13+3.25)/3.25 = 10

Chapter 9: Ptotal = Constant:

| 3> Amy (with mass 55 kg) and George (mass 70 kg) are sitting at opposite ends of a canoe (mass 22 kg) at rest on frictionless and still water. She tosses him a 14 kg pack, giving it a speed of 3.1 m/s relative to the water. What is the speed of the canoe in m/s relative to the water while the pack is in the air

a) 0.27 ->b) 0.30 c) 0.96 d) 3.1

e) All that can be said is: the canoe will move in the same direction as the pack.

$P_i = P_f$ => $0 = mV - (M_A + M_G + M_C)v$ => v = 14(3.1)/(55+70+22) = 0.2952380952

Chapter 9: Impulse:

| 4> A 150 gram baseball is moving horizontally at 30 m/s just before reaching the bat. Bat and ball are then in contact for 2.0 millisecs. Find the force exerted by the bat on the ball if the hit is a line drive to center field (ie the ball was hit directly back from where it was pitched) at 45 m/s?

a) 1,100 N b) 4,500 N ->c) 5,600 N d) 7,500 N e) 23,000 N

$\mathbf{J} = \mathbf{F}_{avg}\Delta t = \Delta\mathbf{p} = \mathbf{P}_f - \mathbf{P}_i = mv_f\mathbf{out} - mv_o\mathbf{in} = m(v_f+v_o)\mathbf{out}$

=> $F = \Delta p/\Delta t = m(v_f+v_o)/\Delta t = (.150)(45+30)/0.002 = 5,6250000$

Chapter 9, Inelastic Collision:

| 5> A hockey coach enlists a physics student on his team to measure the speed of a hockey puck. The student loads a small Styrofoam chest with sand, giving it a total mass of 6.4 kg. He places the chest on frictionless ice, in the path of the hockey puck, whose mass is 160 grams. The puck embeds itself in the Styrofoam, and the chest moves off at a speed of 1.2 m/s. What was the speed of the puck in m/s?

a) 0.29 b) 1.2 c) 7.7 ->d) 49 e) 59

$mv_i = (m+M)V_f$ => $v_i = (m+M)V_f/m = (.16+6.4)1.2/.160 = 49.2000$ m/s

Chapter 9, Elastic Collision: M>>m

| 6> A mass M moves with an initial speed V. M makes a one-dimensional elastic head-on collision with a mass m which is initially at rest. If M = 100m, then it is *exactly* true that: the velocity of ___ after the collision is ____.

a) m; 2V b) M; V c) m; (201/100)V d) M; (101/99)V ->e) m; (200/101)V

$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f} \Rightarrow MV = Mv_{1f} + mv_{2f} = m(100v_{1f} + v_{2f})$

$(v_{1i} - v_{2i}) = -(v_{1f} - v_{2f}) \Rightarrow V = -(v_{1f} - v_{2f}) \Rightarrow v_{2f} = V + v_{1f} \Rightarrow$

$MV/m = 100v_{1f} + V + v_{1f} \Rightarrow (M-m)V/m = 101v_{1f} \Rightarrow 99V = 101v_{1f} \Rightarrow v_{1f} \Rightarrow = V99/101 \approx V$

$v_{2f} = V + v_{1f} = V + 99V/101 = 200V/101 \approx 2V$

Chapter 10: Eqns of Uniformly Angularly Accelerated Motion: $\omega^2 = \omega_o^2 + 2\alpha(\theta - \theta_o)$

| 7> A flywheel starting from rest rotates with a constant angular acceleration of 6.0 rad/s^2. When the wheel has rotated through 4 complete revolutions, its angular velocity is _____ radians/sec.

a) 0.048 b) 6.9 c) 8.7 d) 12 ->e) 17

$\omega^2(t) = \omega_o^2 + 2\alpha(\theta - \theta_o) = 0 + 2(6)[4rev(2\pi Rad/Rev)] = 301.6 \Rightarrow \omega = 17.36643011$

Chapter 10: $I = \Sigma m_i r_i^2$ is the Moment of Inertia or Rotational Inertia

Sample Problem P10.5.1 in Text

| 8> Two particles, each with m = 2.0 kg, are connected to the ends of a 4.0 m long light rigid rod with its center at the origin of an x-axis. The system is free to rotate in the xz-plane about the y-axis. The 'Moment of Inertia' or 'Rotational Inertia' of the pair of masses about the y-axis is _____ kg m^2.

a) 8 ->b) 16 c) 32 d) 64 e) 128

$I = \Sigma m_i r_i^2 = 2(2^2) + 2(2^2) = 16$

Chapter 10: $W_{net} = \Delta K = \frac{1}{2}I\omega^2$

| 9> Denoting the Moment of Inertia of the System in the previous Question by I, the work required to accelerate it from rest to a final angular velocity of ω is

a) Iω b) 2Iω c) $I^2\omega$ d) $I\omega^2$ ->e) $\frac{1}{2}I\omega^2$

$W_{net} = \Delta K = \frac{1}{2}I\omega^2$

Chapter 10, Torque and Angular Acceleration:

|10> If the System in the previous Questions reaches its final angular velocity ω uniformly in a time t, then the constant torque acting is

->a) Iω/t b) Iω/(2t) c) $I\omega^2/t$ d) $I\omega^2/(2t)$ e) $I/(\omega^2t^2)$

$\tau = I\alpha \Rightarrow \tau = I[(\omega-\omega_o)/t] = I\omega/t$

Chapter 11, Conservation of Angular Momentum:

|11> A turntable with I = 0.5 kgm^2 is rotating with an angular velocity ω_o = 10 rad/sec when a blob of putty with m = 2.0 kg falls vertically onto it at a distance of 0.2 m from the axis. The putty sticks to the turntable. Its new angular velocity is ω = _____ rad/sec.

a) 5.5 ->b) 8.6 c) 10 d) 13 e) 63

$I_i\omega_i = I_f\omega_f \Rightarrow \omega_f = I_i\omega_i/I_f = 0.5(10)/[0.5+2(0.2)^2] = 8.620689655$

Chapter 11, Conservation of Angular Momentum:
Chapter 11 Homework Problem, P.11.47:

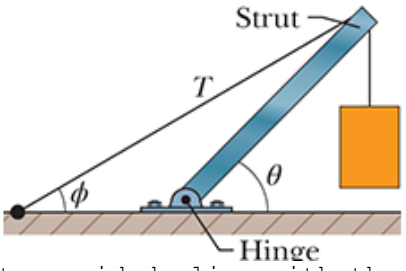
|12> A track is mounted on a large wheel that is free to turn with negligible friction about a vertical axis. A toy train of mass m is placed on the track, and with the system (ie track + train) initially at rest, the train's electrical power is turned on. The train reaches a speed of v_o = 0.20 m/s with respect to the track. What is the angular speed of the wheel in radians/second if its mass is M = 1.5m (where m is the mass of the toy train), and its radius is r = 0.40 meters. Ignore the width of the track.
Treat the wheel as a hoop, neglecting the mass of the spokes and hub.
a) 0.13 b) 0.17 ->c) 0.20 d) 0.33 e) 0.66



r = Radius of Wheel, T = Track, L = Lab
$\tau_{net,ext,sys_z} = 0 \Rightarrow \mathbf{L}_{tot_sys_z}$ = constant $\Rightarrow L_{zi} = L_{zf} \Rightarrow 0 = [\mathbf{r}X(\mathbf{p}=m\mathbf{v})]_z + [I_w\boldsymbol{\omega}]_z$
$\Rightarrow 0 = +mvr + -I\omega_w$ where: $v_{mL} = v_{mT} + v_{TL} \Rightarrow v = v_o - r\omega$
$\Rightarrow m(v_o - r\omega)r = (I = Mr^2)\omega \Rightarrow mv_or = (m+M)r^2\omega \Rightarrow \omega = mv_o/[(m+M)r]$
$\omega = .20/[(1+1.5)0.40] = 0.20$ rad/s

Chapter 12: Equilibrium
Chapter 12 Problem P.12.21:

|13> A concrete block of mass M = 225 kg hangs from the end of a uniform strut of mass m = 45.0 kg. A cable runs from the ground, over the top of the strut, and down to the block, holding it in place. This system is in equilibrium for angles $\phi = 30.0°$ and $\theta = 45.0°$. Find the tension, T, in the cable in Newtons.
->a) 6,630 b) 4,850 c) 3,010 d) 2,650
e) Need to know the Length of the strut!



Choose the Hinge as the origin to compute torques in order to avoid dealing with the Force of the Hinge on the strut at the Hinge which we do not know.
Let L = Length of Strut. Let α = angle between cable and strut. For the Triangle formed by the Strut, the Cable and the floor the sum of their interior angles are:
$180 = (180-\theta) + \phi + \alpha \Rightarrow \alpha = \theta - \phi = 15°$
Net Torque on Strut = $\boldsymbol{\tau}_{net} = 0 = \boldsymbol{\tau}_{net} = \boldsymbol{\tau}_T + \boldsymbol{\tau}_m + \boldsymbol{\tau}_M = +TL\sin\alpha + -mg(L/2)\cos\theta + -MgL\cos\theta$
$+T\sin(\theta - \phi) + -(mg/2)\cos\theta + -Mg\cos\theta = 0 \Rightarrow +T = [(M + m/2)g\cos\theta]/\sin(\theta - \phi)$
T = 6,626.589234
NOTE: The Hinge is fastened to the ground, so the vertical Component of the Force on the strut by the Hinge contains the Normal Force.

BONUS :

|14> What is the magnitude of the Force in Newtons acting on the strut by the Hinge in the Previous Problem?

a) 5,740 b) 5,960 c) 8,120 ->d) 8,270 e) 8,430

$\mathbf{F}_{net_on_strut} = \mathbf{T} + m\mathbf{g} + M\mathbf{g} + \mathbf{F} = 0 \Rightarrow$

$F_{net_horizontal} = -T\cos\phi + F_{horizontal} = 0 \Rightarrow F_{horizontal} = +T\cos\phi = +5{,}738.794617$

$F_{net_vertical} = -T\sin\phi + F_{vertical} - mg - Mg = 0 \Rightarrow F_{vertical} = mg + Mg + T\sin\phi = +5{,}959.294617$

$F = \sqrt{[F_h^2 + F_v^2]} = 8{,}273.267552$ & $\tan(\beta) = F_v/F_h \Rightarrow \beta = 46.07985357°$

A check on the consistency of the above results_____

F = magnitude of the Force on the strut by the Hinge.

Let β = Angle between the floor and the **F** acting on the strut by the Hinge.

Choose the Top-end of the Hinge as the origin to compute torques in order to have 0 torques from the T and Mg forces. Let L = Length of Strut and

Net Torque on Strut = $\boldsymbol{\tau}_{net} = 0 = \boldsymbol{\tau}_{net} = \boldsymbol{\tau}_T + \boldsymbol{\tau}_m + \boldsymbol{\tau}_M + \boldsymbol{\tau}_F = +T0 + +mg(L/2)\cos\theta + +Mg0$

$-F_{vertical}L\cos\theta + +F_{horizontal}L\sin\theta = 155.9170453 - 4213.857635 + 4057.940590 =$

$= -6.7 \text{ X } 10^{-7} = 0$ with round off (I cancelled L in each term.)

Another check on the consistency of the above results_____

Let x = distance from where the cable is tied to the ground to Hinge.

from Law of Sines: $\sin\alpha/x = \sin\phi/L \Rightarrow x = L\sin\alpha/\sin\phi = (0.5176380902)L$

Choose where the cable attaches to the ground as the origin to compute torques in order to have 0 torques from just T.

Net Torque on Strut = $\boldsymbol{\tau}_{net} = 0 = \boldsymbol{\tau}_{net} = \boldsymbol{\tau}_T + \boldsymbol{\tau}_m + \boldsymbol{\tau}_M + \boldsymbol{\tau}_F =$

$= +T0 + -mg[x + (L/2)\cos\theta] + -Mg[x + L\cos\theta] + F_{vertical}[x] + F_{horizontal}[0]$ L cancels.

$= -384.1954430 - 2700.562441 + 3{,}084.757885 = 0$

Yet Another check on the consistency of the above results_____

Net Torque on Strut = $\boldsymbol{\tau}_{net} = 0 = \boldsymbol{\tau}_{net} = \boldsymbol{\tau}_T + \boldsymbol{\tau}_m + \boldsymbol{\tau}_M + \boldsymbol{\tau}_F =$

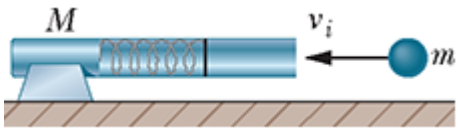
$= +T0 + -mg[x + (L/2)\cos\theta] + -Mg[x + L\cos\theta] + F[x\sin\beta]$ L cancels from each term.

$= -384.1954430 - 2{,}700.562441 + 3{,}084.757884 = 0.000000000$

Problems:

Chapter 9 Homework Problem P.9.57:

|P1> A ball of mass 'm' is shot with speed 'v_o' into the barrel of a spring gun of mass 'M' initially at rest on a frictionless surface. The spring constant of the spring in the barrel of the gun is "k". The ball sticks in the barrel at the point of maximum compression of the spring, "x". The speed of the spring gun after the ball stops in the barrel is "v". Assume that the increase in thermal energy due to friction between the ball and the barrel is negligible. Your final answers should be simplified as much as possible, and should only use the symbols defined in the statement of this problem.



a) Find the speed v in terms of m, M, and v_o. (5 Points)
b) What fraction of the initial kinetic energy of the ball is stored in the spring? Express this fraction in terms of only m and M. (5 Points)
c) Find how much the spring is compressed in terms of m, M, k, and v_o. (5 Points)

a) $mv_o = (m + M)v \Rightarrow v = mv_o/(m + M)$
'x', will be a maximum when the bullet and gun are not moving relative to each other which tells us that at that moment the speed of the bullet is the same as the speed of the gun which will be symbolized by 'v'.

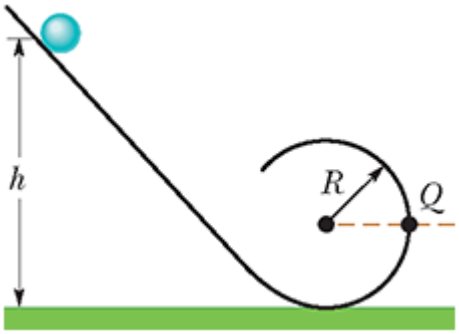
b) $[\frac{1}{2}mv_o^2 - \frac{1}{2}(m+M)v^2]/\frac{1}{2}mv_o^2 \Rightarrow [\frac{1}{2}mv_o^2 - \frac{1}{2}(m+M)\{mv_o/(m+M)\}^2]/\frac{1}{2}mv_o^2 \Rightarrow 1 - m/(m+M) = M/(m+M)$

c) The Kinetic Energy Lost = the Potential Energy stored in the Spring by ...
Conservation of Mechanical Energy: $E_i = E_f \Rightarrow K_i + U_i = K_f + U_f$
$\frac{1}{2}mv_o^2 = \frac{1}{2}(m+M)v^2 + \frac{1}{2}kx^2 \Rightarrow \frac{1}{2}kx^2 = [\frac{1}{2}mv_o^2 - \frac{1}{2}(m+M)v^2]$
$x^2 = \{mv_o^2 - (m+M)[v = mv_o/(m + M)]^2\}/k = \{mv_o^2 - (mv_o)^2/(m + M)]\}/k$
$x^2 = mv_o^2\{1 - (m/(m + M)]\}/k = mv_o^2\{[(m+M)-m]/(m + M)]\}/k = mv_o^2\{[M/(m + M)\}/k$
$x^2 = mMv_o^2/[k(m + M)] \Rightarrow x = \sqrt{\{mMv_o^2/[k(m + M)]\}}$

Check: $\frac{1}{2}mv_o^2 = \frac{1}{2}(m+M)v^2 + \frac{1}{2}kx^2 \Rightarrow \frac{1}{2}mv_o^2 = \frac{1}{2}(m+M)[mv_o/(m + M)]^2 + \frac{1}{2}k\{mMv_o^2/[k(m + M)]\}$
$\frac{1}{2}mv_o^2 = \frac{1}{2}[mv_o]^2/(m + M) + \frac{1}{2}mMv_o^2/(m + M) = \frac{1}{2}mv_o^2\{m/(m + M) + M/(m + M)\}$ checks!

Chapter 11 Homework Problem, P.11.12 except r is not very small:

|P2> A solid brass ball of mass 'm' and radius 'r' will roll without slipping along the loop-the-loop track when released from rest at height 'h' along the straight section (h is the height of the ball's center of mass above the floor). The circular loop has radius 'R'. In your analysis below use a y-axis with origin (ie y≡0) at the floor level. The initial position of the ball's center of mass is then "y" = h. Assume that there is no friction. Do NOT assume that r<<R as in your homework problem. Let 'v' symbolize the speed of the ball's center of mass at the top of the loop. Your Equations should only use the constants: m, r, R, h, g, and the variables, v (speed of ball), and y (height of Center of Mass of ball).



a) Assuming the ball does not leave the track, write an equation based on Energy Considerations that will determine the speed of the ball's center of mass, v, when it is at the top of the loop? (9 Points)

b) Write out the Equation of Motion (ie Newton's 2nd Law) for the ball at the top of the trajectory for the case where the ball is on the verge of leaving the track at the top of the loop. (6 Points)

c) Use the results of parts (a) & (b) to determine the height h that the ball needs to be released from in order that it is on the verge of leaving the track at the top of the loop. (5 Points)

a) $E = \text{Constant} \Rightarrow E_i = E_f \Rightarrow mgy_{cmi} = \frac{1}{2}mv^2 + \frac{1}{2}I_{CM}\omega^2 + mgy_{cmtop}$
$mgh = \frac{1}{2}mv^2 + \frac{1}{2}(2mr^2/5)(v^2/r^2) + mg(2R-r)$
$gh = \frac{1}{2}v^2 + v^2/5 + g(2R-r) \Rightarrow g(h+r-2R) = 7v^2/10 \Rightarrow v^2 = 10g(h+r-2R)/7$
$v = \sqrt{[10g(h+r-2R)/7]}$

b) $\mathbf{F}_{net} = m\mathbf{a} \Rightarrow \mathbf{F}_{centripetal} = m\mathbf{a}_{centripetal} \Rightarrow [mg + N\equiv 0](-\mathbf{j}) = [mv^2/(R-r)](-\mathbf{j}) \Rightarrow$
$v^2 = (R-r)g$

c) $v^2 = 10g(h+r-2R)/7 = \text{also} = g(R-r) \Rightarrow 10(h+r-2R) = 7(R-r) \Rightarrow 10h = (20+7)R - (10+7)r$
$h = 27R/10 - 17r/10 = 2.7R - 1.7r$