Calculus-based Physics 201
Test #1:
Measurement, Vectors, & Kinematics
Fall 2026

Numerical Answers: Use the MKS Metric System unless explicitly told to do otherwise.
Determine Numerical Answers to at least Three Significant Figures.
NO: Notes, Formula Cards, Formulas or Constants in your Calculators, or any other aids.
Vectors are indicated in Bold Print (**v**).
Unit Vectors in the x, y, & z directions are symbolized by **i**, **j**, & **k** respectively.
Multiple Choice Questions (and BONUS) are 5 points each, and the Problems are 20 points.

Chapter 1: Units & amu & Avogadro's #:

| 1> The penny was officially discontinued on November 12th, 2025. A short history of the Lincoln Cent follows. First minted in 1909 to mark the Centennial of Lincoln's birth, and until 1982 had a mass of 3.11 grams, 95% of which was copper (ie 2.95 grams of copper) and 5% zinc (or a mix of zinc and tin depending on the decade). To cut costs in the middle of 1982 the composition of the penny changed to 97.5% zinc with a 2.5% copper coating; the new penny's mass was only 2.5 grams. Your problem is to find how many atoms of copper were in the pre-1982 Lincoln Cent? Copper has Atomic Number 29, and assume there are 34 neutrons in its nucleus.

->a) 2.8×10^{22} b) 3.1×10^{22} c) 5.3×10^{22} d) 6.2×10^{22} e) 5.2×10^{25}

Mass of Cu atom in amu: 29 + 34 = 63 amu.
Mass of Cu atom in grams = 63 amu$(1 \text{ gram}/N_A \text{ amu}) = 1.046164065 \times 10^{-22}$ = call this m
Mass of Cu in a Penny = M = (N = # of Cu atoms in a penny)(m = mass of a Cu Atom) => N = M/m = 2.95/m = $2.819825397 \times 10^{+22}$.
Using 0.95(3.11) = 2.9545000000 => N = $2.824126825 \times 10^{22}$.

Chapter 2: Instantaneous Velocity = dy/dt:

| 2> The altitude of the space shuttle in the first half-minute of its ascent is given by $y = bt^2$, where the constant b is 2.90 m/s^2, y is in meters, and t is the time in seconds since lift-off. Assuming that the shuttle is moving only in the vertical direction (the +y direction), what is the instantaneous velocity of the shuttle at t = 20.0 sec.

a) 54 m/s b) 60 m/s ->c) 116 m/s d) 174 m/s e) 7,700 m/s

$v = dy/dt = 2bt = 2(2.9)20 = 116$ m/s

Chapter 2: $y(t) = y_o + v_o t - \frac{1}{2}gt^2$ or **$h = \frac{1}{2}gt_f^2$:**

| 3> The fountains of lava at a volcanic eruption are so hot that even geologists don't get near them. But they can watch a chunk of lava that has been thrown upward, and time its fall. Lava chunks at the top of the fountain take 5.0 seconds to fall. How high are the fountains?

a) 490 m ->b) 120 m c) 49 m d) 25 m
e) Need the time necessary for the lava to get to the top of the fountain.

time to go up = time to fall down = t_f => $h = \frac{1}{2}gt_f^2 = 122.5000000...$ m

Chapter 2: $a=<a>=\Delta v/\Delta t$ & $v^2(t) = v_o^2 + 2a(x-x_o)$ find a:

| 4> I was riding my bicycle at a leisurely rate of 10 km/hr, then I sped up at a steady rate for 10 seconds to escape a dog. At the end of that time, I was doing 30 km/hr. How far did I go during the 10 second time interval?

a) Need to be given the acceleration.
b) 5.00 m c) 28 m ->d) 55.6 m e) 62.5 m

Find a and then use $v^2(t) = v_o^2 + 2a(x-x_o)$
$a = \Delta v/\Delta t = (30-10)/10 = 2$km/hr/s$(1\text{hr}/3600\text{s})(1000\text{m/km}) = 0.55555555555...$ m/s^2
$v = 30$ km/hr$[(1000\text{m/km})(1\text{hr}/3600\text{s})] = 8.33333333\ldots$ m/s
$v = 10$ km/hr$[(1000\text{m/km})(1\text{hr}/3600\text{s})] = 2.77777777\ldots$ m/s
$(x-x_o) = (v^2 - v_o^2)/(2a) = 55.5555...$ m

Chapter 2: Average Speed

| 5> A car travels a certain distance, d, at an Average Speed of 80 km/hr, and then travels the same distance, d, at an Average Speed of 40 km/hr.
The Average Speed of this car for the trip of distance 2d is what in km/hr?

a) 60 ->b) 53 c) 48 d) 45 e) 42

Average Speed ≡ {total distance travelled}/{total time to travel}
=> {total distance travelled} = [Average Speed]{total time to travel}
=> {total time to travel} = {total distance travelled}/[Average Speed]
time to travel at average speed: $\Delta t1 = d/\langle v1\rangle$ & $\Delta t2 = d/\langle v2\rangle$
Average Speed ≡ {2d}/($\Delta t1 + \Delta t2$} = {2d}/(d/$\langle v1\rangle$ + d/$\langle v2\rangle$} = {2}/(1/$\langle v1\rangle$ + 1/$\langle v2\rangle$}
= 2/{1/40 + 1/80}[80/80] = 160/(2 + 1) = 53.33333333333333

Chapter 2: $x(t) = x_o + v_o t + \frac{1}{2}at^2 \Rightarrow x_1(t_f) = x_2(t_f)$ find t_f then $x(t_f)$:

| 6> A speeding motorist zooms through a 50 km/hr zone at 75 km/hr without noticing a stationary police car by the roadside. The police officer immediately heads after the speeder accelerating at the constant rate of 9.0 km/hr/s. When the officer pulls alongside the speeder, how far down the road are they?

a) 1300 m ->b) 350 m c) 170 m d) 154 m e) 85 m

v_{om} = 75 km/hr[(1000m/km) (1hr/3600s)] = 20.83333333… m/s
$a_p = \Delta v/\Delta t$ = 9.00 km/hr/s(1hr/3600s)(1000m/km)= 2.50000000... m/s^2
$x_m(t) = x_{om} + v_{om}t + \frac{1}{2}a_m t^2 = 0 + v_{om}t + 0$ & $x_p(t) = x_{op} + v_{op}t + \frac{1}{2}a_p t^2 = 0 + 0 + \frac{1}{2}a_p t^2$
$x_m(t) = x_p(t) \Rightarrow v_{om}t = \frac{1}{2}a_p t^2 \Rightarrow 2v_{om}/a_p = t$ = 16.6666666... s
$x_m(t=2) = v_{om}t$ = 347.22222222...m ~350m

Chapter 3: Displacement Vector:

| 7> You leave Los Angeles on a flight to Denver, with a stop in Salt Lake City. Flying 55° North of East, the plane makes the 970 km trip to Salt Lake City in 72 minutes. After 40 minutes on the ground, the plane heads for Denver, which is 600 km away and in a direction 10° South of East. The plane reaches Denver 55 minutes after leaving Salt Lake City. Determine the net displacement vector for the whole trip. (The x-axis is pointing East, and the y-axis points to the North.)

a) 1570 km 31° North of East b) (34**i** + 690**j**) km c) (690**i** + 1147**j**) km
d) (1147**i** + 899**j**) km ->e) (1147**i** + 690**j**) km

$\mathbf{d} = \mathbf{d}_1 + \mathbf{d}_2$ = [970cos(55)**i** + 970sin(55)**j**] + [600cos(-10)**i** + 600sin(-10)**j**]
[970cos(55)**i** + 970sin(55)**j**] + [600cos(10)**i** + -600sin(10)**j**]
= (556.3691433+590.8846518)i + (794.5774830-104.1889066)j]
= 1147.253795i + 690.3885764j

NOTE: keep all quantities used in answering the question below because they can be used to help answer the next question.

Chapter 3: Scaler Product:

| 8> Two vectors are given by **A** = 1.23**i** + 2.31**j** + 3.12**k** and **B** = -4.56**i** + 5.64**j** + -6.45**k**. What is the angle between **A** & **B** in degrees?

a) 18.7° b) 97.3 c) 105 ->d) 109 e) 111

$\mathbf{A}\cdot\mathbf{B} = AB\cos\theta$ & $\mathbf{A}\cdot\mathbf{B} = AxBx + AyBy + AzBz$ and $A = \sqrt{[Ax^2 + Ay^2 + Az^2]}$
$\mathbf{A}\cdot\mathbf{B}$ = (1.23)(-4.56) + (2.31)(5.64)+ (3.12)(-6.45) = -12.7044000
$A = \sqrt{[1.23^2 + 2.31^2 + 3.12^2]}$ = 4.072272093 & B = 9.705962085
$\cos\theta = \mathbf{A}\cdot\mathbf{B}/AB$ = -0.3214243546 => θ = 108.7490857°

Chapter 3: Vector Product:

| 9> What is the magnitude of **A**X**B** where **A** & **B** are given in the previous question?

a) 12.7 b) 36.9 ->c) 37.4 d) 38.2 e) 39.2

$|\mathbf{A}\mathbf{X}\mathbf{B}| = AB\sin\theta$ = 37.42791774

Chapter 3: Vector Product:

|10> What is the component of **A**X**B** (from the previous question) in the direction of the vector **C** = 7.89**i** + -8.97**j** + 9.78**k**?

a) -0.736 b) +29.1 c) -29.1 d) +1.88 ->e) -1.88

AX**B** = **x**(AyBz - AzBy) + **y**(AzBx - AxBz) + **z**(AxBy - AyBx)
AX**B** = **x**[(2.31)(-6.45) - (3.12)(5.64)] + **y**[(3.12)(-4.56) - (1.23)(-6.45)] + ...
z[(1.23)(5.64) - (2.31)(-4.56)]
AX**B** = **x**(-32.4963000) + **y**(-6.2937000) + **z**(17.4708000)
$C = \sqrt{[7.89^2 + -8.97^2 + 9.78^2]}$ = 15.43895722
A unit vector in the direction of C is **u** = **C**/C
The component of **A**X**B** in the direction of **C** is (**A**X**B**)·**u** = (**A**X**B**)·**C**/C =
= [(-32.4963000)7.89 + (-6.2937000)(-8.97) + (17.4708000)9.78]/15.43895722 =
= -29.07689400/15.43895722 = -1.883345720

Chapter 4: projectile motion:

|11> An object slides with a speed of v_o = 2.5 m/s off of a horizontal table top that is 1.5 m above the ground. How far from the edge of the table does it land?

->a) 1.38 m b) 0.38 m c) 1.76 m d) 0.76 m e) 1.43 m

NOTE: $v_{oy} \equiv 0$ => $h = \frac{1}{2}gt_f^2$ & Uniform Motion with $a_x \equiv 0$ => $x = (x_o = 0) + (v_{ox} = v_o)t_f$ => $x = v_o\sqrt{[2h/g]} = 1.383208338$

Chapter 4: $\mathbf{a} \equiv d\mathbf{v}/dt = \mathbf{a}_t + \mathbf{a}_c = (dv/dt)\mathbf{t} + [v^2/\rho_c]\mathbf{n} = \perp$ Decomposition:

|12> A road makes a 90° bend with a radius of curvature of 190 m. A car enters the bend moving at 20 m/s. Finding this too fast, the driver applies the brakes and decelerates continuously at 0.92 m/s^2 (a result of decreasing his speed; I called this tangential acceleration in class). Determine the magnitude of the total acceleration of the car in m/s^2 when its speed rounding the bend has dropped to 15 m/s.

a) 0.745 b) 0.92 c) 1.18 ->d) 1.50 e) 2.30

$a_c = v^2/r = 15^2/190 = 1.184210526$ => $a = \sqrt{[a_t^2 + a_c^2]} = a = \sqrt{[0.92^2 + 1.184^2]} = 1.50$

$a = 1.499584799$

Chapter 4: $a_c = v^2/r = (2\pi r/T)^2/r$:

|13> A space shuttle is in a circular orbit about the Earth at an altitude of 250 km above Earth's surface. The shuttle is travelling at a constant speed and its acceleration is 93% of the acceleration due to gravity on the Earth's surface (ie a = 0.93g). What is the period of the shuttle's orbit (ie the time to complete one orbit). (The radius of the Earth is 6.37 X 10^6 m.)

a) 0.69 s b) 17 mins. ->c) 89 mins. d) 4.66 hrs e) 1.77 X 10^{10} s.

$r = h + R = 250{,}000 + 6.37 \times 10^{+6} = 6.62 \times 10^{+6}$

$a_c = v^2/r = (2\pi r/T)^2/r = 4\pi^2 r/T^2$ => $T^2 = 4\pi^2 r/a = 4\pi^2(h+R)/[0.93(9.8)] = 28{,}675{,}348.31$

$T = 5{,}354.936817$ s (1 min/60 s) = 89.24894695 mins

|P1> A world class long jumper jumps R = 8.0 m horizontally in the direction of an x-axis where $x(t=0) = 0$ and $x(t_f) = R$. Assuming his horizontal velocity component is $v_{ox} = 9.0$ m/s as he leaves the ground at t=0, find the following ... (show all work using symbols first, then substitute in numerical values to get final numerical results)
NOTE: We are treating the jumper as a Particle that takes off from the ground and lands on the ground at the same vertical height.

a) The total time he is in the air, ie t_f. (5 Points)

$x(t_f) = R = v_{ox}t_f \Rightarrow t_f = R/v_{ox} = 8/9 = 0.88888888$ s

I hope you do not use the Range Formula unless you prove it, but $R = v_o^2 \sin 2\theta_o / g$
It does not help much in this problem since v_o and θ_o are unknowns at this point.

b) His initial vertical velocity component (y-component of velocity), v_{oy}. (5 Points)

He reaches his highest point at the midpoint of his trajectory.
The time to go up is equal to the time to come down $\Rightarrow t_{up} = t_f/2 = 4/9$ s $= 0.44444$
$v_y = v_{oy} - gt_{up} = 0$ at the top of the trajectory.
$\Rightarrow v_{oy} = gt_{up} = 4.3555555$ m/s; and NOTE: $t_{up} = v_{oy}/g$

or: $y(t_f) = (y_o=0) + v_{oy}t_f - \frac{1}{2}gt_f^2 = 0 \Rightarrow v_{oy} = gt_f/2$ which checks with above

c) How high he gets above the ground, $y_{max} = h$. (5 Points)

$v_y^2 = v_{oy}^2 - 2g(y-y_o)$ at the top of the trajectory $v_y \equiv 0 \Rightarrow v_{oy}^2 - 2g(y_{max}-0) = 0$
$\Rightarrow y_{max} = v_{oy}^2/(2g) = 0.9679012346$ s

or: $y_{max} = y(t_{up}) = (y_o=0) + v_{oy}t_{up} - \frac{1}{2}gt_{up}^2 = v_{oy}(v_{oy}/g) - \frac{1}{2}g(v_{oy}/g)^2 = v_{oy}^2/(2g)$ check

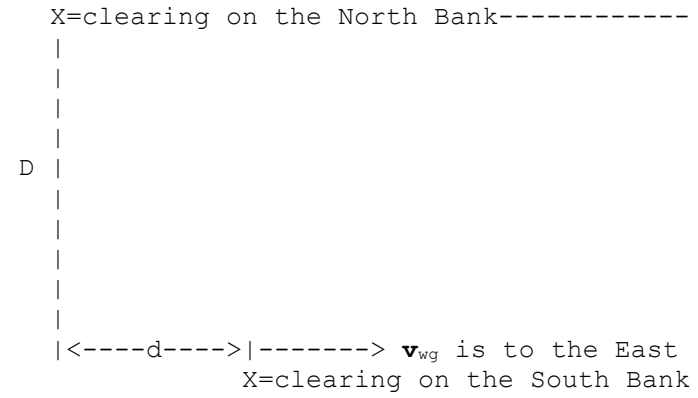
o: $h = ymax = \frac{1}{2}gt_{up}^2 = \frac{1}{2}g(v_{oy}/g)^2 = v_{oy}^2/(2g)$ checks again

d) The angle at which he leaves the ground, θ_o, when he jumps at t=0. θ_o is measured counterclockwise from the positive x-axis. (5 Points)

$\mathbf{v}_o$ is in the 1st quadrant $\Rightarrow \tan\theta_o = v_{oy}/v_{ox} = g(4/9)/9 = 4g/81 = 0.4839506173$
$\Rightarrow \theta_o = \tan^{-1}(v_{oy}/v_{ox}) = 25.82468967°$

|P2> A D = 234-m-wide river has a uniform velocity of $\mathbf{v}_{wg}$ = 1.2 m/s through a jungle and toward the East. An explorer wishes to leave a small clearing on the South bank and cross the river in a powerboat that moves at a constant speed of v_{bw} = 4.5 m/s with respect to the water. There is a clearing on the North bank d= 82 m upstream from a point directly opposite the clearing on the South bank. Each part below is 5 Points.

a) Using the Symbolic Notation defined in the statement of the Problem draw a *Velocity Addition Diagram* for the following velocities: $\mathbf{v}_{bw}$ = velocity of boat wrt (with respect to) water; $\mathbf{v}_{wg}$ = velocity of water wrt ground; and $\mathbf{v}_{bg}$ = velocity of boat wrt ground. I have already drawn the velocity $\mathbf{v}_{wg}$ in the Diagram below. Let the angles opposite the sides $\mathbf{v}_{wg}$, $\mathbf{v}_{bw}$, and $\mathbf{v}_{bg}$ be called β, γ, & α respectively.



b) Determine the *Heading* of the boat, θ, where *Heading* is the direction the boat must be pointed in order to travel a straight line and land in the clearing on the North bank. The Direction Angle, θ, is measured counterclockwise relative to due East which you can take to be the direction of an x-axis. Your analysis should be written entirely with the use of the symbols defined here. If you need any additional Symbols, then clearly define them. Only insert numbers in the Last Step to get a numerical value for θ. Show work below statement of Part (c).

c) Determine $\mathbf{v}_{bg}$: both its magnitude v_{bg} and angle direction measured counterclockwise from due East. As in (b) analyze in symbols getting a final numerical value in the Last Step. Note that $\mathbf{v}_{bg}$ is in the direction of the *Bearing* of the boat. Bearing is the direction from the ship to its destination.

NOTE that $180-\gamma = \delta$ is such that $\tan\delta = D/d$ =>
$\delta = \tan^{-1}(D/d = 234/82) = 70.068814344$ => $\gamma = 180 - \delta = 109.3118566°$
Law of Sines: $vbw/\sin\gamma = vwg/\sin\beta$ => $\sin\beta = (vwg/vbw)\sin\gamma = 0.2516620096$
=> $\beta = 14.57588317°$
NOW since $\alpha + \beta + \gamma = 180$ => $\alpha = 180 - \beta - \gamma = 56.11226027°$
Heading = $\theta = 180-\alpha = 123.8877397°$

Law of Cosines: $vbg^2 = vbw^2 + vwg^2 - 2v_{bw}v_{wg}\cos\alpha$ => $vbg = 3.958316703$
$\mathbf{v}_{bg}$: magnitude = 3.958316703 Direction = $\alpha = 180 - \beta - \gamma = 56.11226027°$

or
Law of Sines: $vbg/\sin\alpha = vwg/\sin\beta = vbw/\sin\gamma$ => $vbg = vwg[\sin\alpha/\sin\beta] = 3.958316703$
or
Law of Sines: $vbg/\sin\alpha = vwg/\sin\beta = vbw/\sin\gamma$ => $vbg = vbw[\sin\alpha/\sin\gamma] = 3.958316703$

d) Let r_{bg} represent the distance (magnitude of displacement vector $\mathbf{r}_{bg}$) the boat travels across the river. Find the time Δt that the boat takes to cross the river and land in the clearing. Symbolically analyze, then get final numerical answer.

$r_{bg}^2 = D^2 + d^2$ => $r_{bg} = 247.9516082$
$r_{bg} = v_{bg}\Delta t$ => $\Delta t = r_{bg}/v_{bg} = 62.64066945$ s