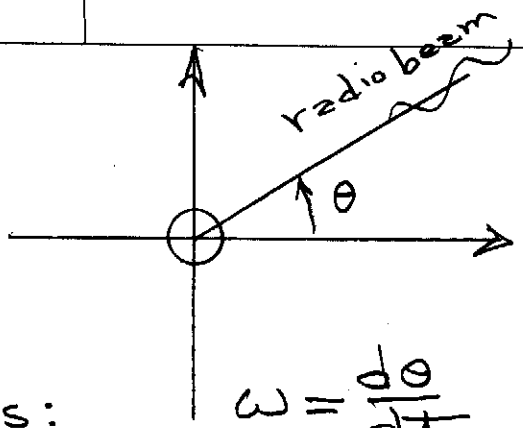
STAEDTLER® No. 937 811E Engineer's Computation Pad

•••32 A pulsar is a rapidly rotating neutron star that emits a radio beam the way a lighthouse emits a light beam. We receive a radio pulse for each rotation of the star. The period T of rotation is found by measuring the time between pulses. The pulsar in the Crab nebula has a period of rotation of $T = 0.033$ s that is increasing at the rate of 1.26×10^{-5} s/y. (a) What is the pulsar's angular acceleration α? (b) If α is constant, how many years from now will the pulsar stop rotating? (c) The pulsar originated in a supernova explosion seen in the year 1054. Assuming constant α, find the initial T.



The Angular Velocity is: $\omega = \frac{d\theta}{dt}$
which is decreasing in time, however this decrease is very slow relative to the Period of the Motion $= T = \frac{1}{30}$ sec.
Therefore we may write $\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{T}$,
$\omega(T)$ is decreasing in time.
The Angular Acceleration is $\alpha = \frac{d\omega}{dt}$
We have $\omega = \omega(T)$ but $T = T(t)$ is increasing at the rate of 1.26×10^{-5} s/y which tells us

$$\frac{dT}{dt} = 1.26\times10^{-5}\ \frac{\text{sec}}{\text{year}}\left(\frac{1\text{ year}}{365.25\text{ days}}\right)\left(\frac{1\text{ day}}{24\text{ hr}}\right)\left(\frac{1\text{ hr}}{3600\text{ s}}\right)$$

$$= 3.99\times10^{-13}\ (\text{unitless})\ (\text{it slows by } 4\times10^{-13} \text{ of a unit per unit})$$

We know that there is a function $T = T(t)$
Therefore $\omega = \omega(t) = \omega(T(t)) \Rightarrow$ use the Chain Rule: $\alpha \equiv \frac{d\omega}{dt} = \frac{d\omega}{dT}\frac{dT}{dt} = \frac{-2\pi}{T^2}\frac{dT}{dt}$

a) $\alpha = \frac{-2\pi}{(1/30)^2}(4.0\times10^{-13}) = -2.3\times10^{-9}$ rad/s^2

$\alpha = \frac{d\omega}{dt} \overset{\text{constant}}{=\!=} \frac{\Delta\omega}{\Delta t} \Rightarrow \omega = \omega_0 + \alpha t$ (UNIFORMLY ANGULAR ACCELERATION)

b) $\Rightarrow t = \frac{\omega - \omega_0}{\alpha} = \frac{0 - 2\pi/1/30}{\alpha} = \frac{-60\pi}{-2.3\times10^{-9}} = 82\times10^9$ years

$\omega\,(\overset{2007}{\text{TODAY}}) = \omega_0\,(1054 \text{ WHEN BORN}) + \alpha t\,(= 2007 - 1054 = 953 \text{ years})$

$\omega_0 = \frac{2\pi}{T = 1/30} - \left[\alpha t = -2.3\times10^{-9}\ \frac{\text{rad}}{\text{s}}\ 953\text{ years}\ \frac{3.16\times10^7\text{ s}}{\text{year}}\right] = 258\ \frac{\text{rad}}{\text{s}}$

$\omega_0 = 258\ \frac{\text{rad}}{\text{s}} = \frac{d\theta}{dt} = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{T_0} \Rightarrow T_0 = \frac{2\pi}{\omega_0} = 0.0245 = \frac{1}{41}$ s