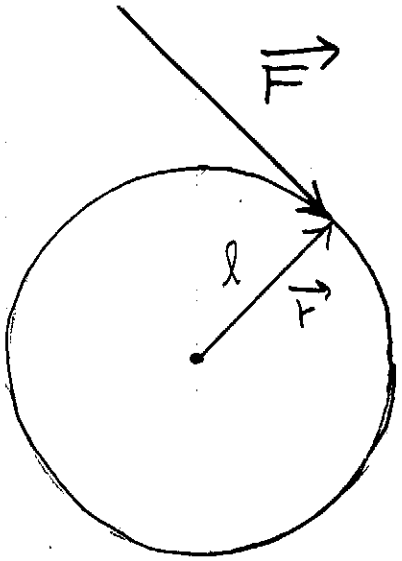
STAEDTLER® No. 937 811E Engineer's Computation Pad

**•••57** A pulley, with a rotational inertia of $1.0 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$ about its axle and a radius of 10 cm, is acted on by a force applied tangentially at its rim. The force magnitude varies in time as $F = 0.50t + 0.30t^2$, with $F$ in newtons and $t$ in seconds. The pulley is initially at rest. At $t = 3.0$ s what are (a) its angular acceleration and (b) its angular speed?




$\vec{\tau}_{net} = I\vec{\alpha}$

$\tau_{net} = |\vec{r} \times \vec{F}| = Fl$

$l$ = MOMENT ARM = LEVER ARM

$= r\sin\measuredangle(\vec{r}, \vec{F}) = r\sin 90°$

$= r$ = radius of PULLEY

Direction is INTO THE PAGE

$\tau = Fr = I\alpha \Rightarrow \alpha = \frac{Fr}{I} = \frac{r}{I}\left(0.5t + 0.3t^2\right)$

$\alpha \underset{3s}{=} \left(\frac{r}{I} = \frac{0.1}{0.001}\right)\left(0.5(3) + 0.3(3^2)\right)$ $\alpha$ IS NOT CONSTANT!

$= 100(4.2) = 420\ \text{rad/s}^2 = 4.2 \times 10^2\ \text{rad/s}^2$

$\alpha = \frac{d\omega}{dt} \Rightarrow d\omega = \alpha\, dt \Rightarrow \int_0^{\omega_f} d\omega = \int_0^{t_f} \alpha\, dt$

$\int_0^{\omega_f} d\omega = \omega\Big|_0^{\omega_f} = \omega_f - 0 = \omega_f$

$\int_0^{t_f} \alpha\, dt = \int_0^3 \frac{r}{I}\left(0.5t + 0.3t^2\right) dt = \frac{r}{I}(0.5)\int_0^3 t\, dt$

$+ \frac{r}{I}(0.3)\int_0^3 t^2\, dt = \frac{r}{I}\left\{0.5\,\frac{t^2}{2}\Big|_0^3 + 0.3\,\frac{t^3}{3}\Big|_0^3\right\}$

$= \left(\frac{r}{I} = 100\right)\left\{0.5\,\frac{9}{2} + 0.3\,\frac{27}{3}\right\} = 10\left\{2.25 + 2.7\right\}$

$= 495\ \frac{\text{rad}}{\text{s}} \sim 500\ \frac{\text{rad}}{\text{s}} = 5.0 \times 10^2\ \frac{\text{rad}}{\text{s}}$