STAEDTLER® No. 937 811E Engineer's Computation Pad

•••66 A uniform spherical shell of mass $M = 4.5$ kg and radius $R = 8.5$ cm can rotate about a vertical axis on frictionless bearings (Fig. 10-48). A massless cord passes around the equator of the shell, over a pulley of rotational inertia $I = 3.0 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$ and radius $r = 5.0$ cm, and is attached to a small object of mass $m = 0.60$ kg. There is no friction on the pulley's axle; the cord does not slip on the pulley. What is the speed of the object when it has fallen 82 cm after being released from rest? Use energy considerations.

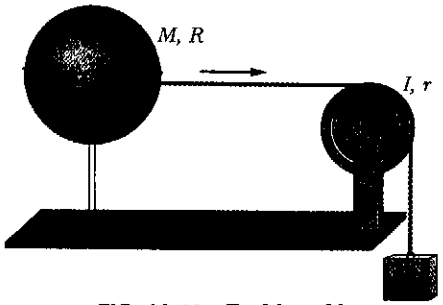


FIG. 10-48 Problem 66.

$\Delta E_{mechanical} = 0$

$E_i = E_f$

$E = K_m + K_M + K_{pulley} + U_M + U_m + U_{pulley}$

$\Rightarrow \Delta K_m + \Delta K_M + \Delta K_{pulley} + \Delta U_M + \Delta U_m + \Delta U_{pulley} = 0$

Let i = @ rest f = arbitrary

$[\Delta K_m = \frac{1}{2} m v^2 - 0] + [\Delta K_M = \frac{1}{2} I_{com}^s \omega_s^2 - 0]$

$+ [\Delta K_{pulley} = \frac{1}{2} I_{com}^p \omega_p^2 - 0] + [\Delta U_M \equiv 0] + [\Delta U_m = \Delta(mgy)$

$+ [\Delta U_{pulley} \equiv 0] = 0$

The STRING HAS AN INSTANTANEOUS SPEED = v = v of m = v (of a pt on rim of pulley) = v (of a pt on Equator of Sphere)

$\Rightarrow \omega_s = \frac{v}{R}$ & $\omega_p = \frac{v}{r}$

$\frac{1}{2} m v^2 + \frac{1}{2}\left(\frac{2}{3} M R^2\right)\frac{v^2}{R^2} + \frac{1}{2} I \frac{v^2}{r^2} + mg\left(\Delta y = y_f - y_i = -h\right) = 0$

$v^2\left\{m + \frac{2}{3}M + \frac{I}{r^2}\right\} = 2mgh$

$v = \sqrt{\frac{2mgh}{m + \frac{2M}{3} + \frac{I}{r^2}}} = \sqrt{\frac{2(0.60)9.80(0.82)}{0.60 + \frac{2(4.5)}{3} + \frac{0.003}{0.05^2}}}$

$= 1.4\ \frac{m}{s}$