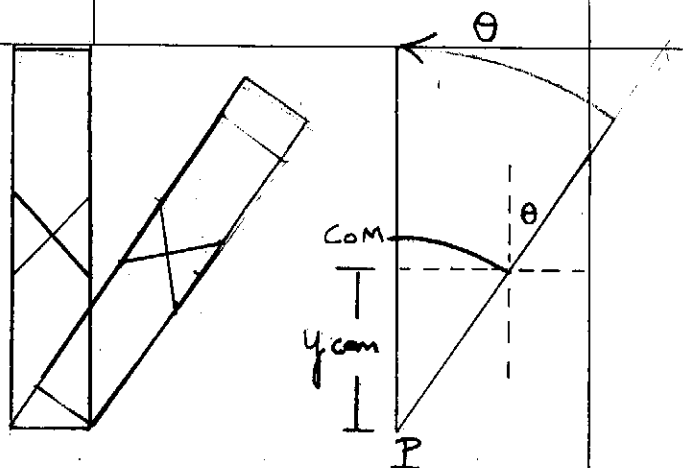
STAEDTLER® No. 937 811E Engineer's Computation Pad

•••67 A tall, cylindrical chimney falls over when its base is ruptured. Treat the chimney as a thin rod of length 55.0 m. At the instant it makes an angle of 35.0° with the vertical as it falls, what are (a) the radial acceleration of the top, and (b) the tangential acceleration of the top. (*Hint:* Use energy considerations, not a torque.) (c) At what angle θ is the tangential acceleration equal to g?



THE GRAVITATIONAL POTENTIAL ENERGY OF THE "RIGID BODY = SYSTEM OF PARTICLES" CAN BE COMPUTED AS IF ALL THE MASS WAS CONCENTRATED AT THE CENTER OF MASS — LOCATED SYMMETRICALLY AT THE CENTER OF THE THIN ROD @ $h/2$ above base WHERE = h = HEIGHT OF CHIMNEY

$$U = mg\,y_{com} = mg\frac{h}{2}\cos\theta$$

$$\Delta E_{mech} = 0 \Rightarrow E_i = E_f \Rightarrow K_i + U_i = K_f + U_f$$

$$\Rightarrow 0 + mg\frac{h}{2} = \frac{1}{2} I_P \omega_P^2 + mg\frac{h}{2}\cos\theta$$

THIS IS UNIQUE TO MY SOLN

THE TEXT WOULD START HERE

SINCE $K = \left(K^{com}_{TRANSLATION} = \frac{1}{2} m v_{com}^2\right) + \left(\frac{1}{2} I_{com}\omega_{com}^2 = K^{com}_{ROTATIONAL}\right)$

$$K = \frac{1}{2} m\left(v_{com} = \frac{h}{2}\omega_P\right)^2 + \frac{1}{2} I_{com}\left(\omega_{com} = \frac{d\theta}{dt} = \omega_P\right)^2$$

$$K = \frac{1}{2}\left\{I_P = I_{com} + m\left(\frac{h}{2}\right)^2\right\}\omega_P^2 \;;\; I_P = \frac{1}{12}mh^2 + m\frac{h^2}{4} = \frac{mh^2}{3}$$

THIS IS THE $I_P = I_{com} + m(\text{dist from com})^2$ THAT WOULD BE OBTAINED FROM THE PARALLEL AXIS THEOREM

$$\Rightarrow \left(\omega_P = \frac{d\theta}{dt}\right)^2 = mgh(1-\cos\theta) \Big/ \left\{I_P = \frac{1}{3}mh^2\right\} = \frac{3g(1-\cos\theta)}{h}$$

RADIAL ACCELERATION @ RADIUS = r: $a_r = a_c = \frac{v^2}{r} = \frac{(r\omega_P)^2}{r} = r\omega_P^2$

a) $a_r = \frac{3gr}{h}(1-\cos\theta)$ & $a_r(\text{@top: } r=h) = 3g(1-\cos\theta) \overset{@35°}{=\!=} 5.32\ m/s^2$

TANGENTIAL ACCELERATION = $a_t = r\alpha$ & $\alpha \equiv \frac{d\omega}{dt}$ = NOT CONSTANT

$$\alpha = \frac{d\omega}{dt} = \frac{d}{dt}\sqrt{\frac{3g}{h}(1-\cos\theta)} = \frac{1}{2}\left[\frac{3g}{h}(1-\cos\theta)\right]^{-1/2}\left[\frac{-3g}{h}(-\sin\theta)\frac{d\theta}{dt}\right]$$

$$\alpha = \frac{1}{2}\frac{1}{\omega}\frac{3g}{h}\sin\theta\,\omega = 3g\sin\theta/2h \Rightarrow a_t = r\alpha = 3gr\sin\theta/2h$$

b) $a_t(\text{@top @35°}) = 3g\sin 35°/2 = 8.43\ m/s^2$

c) $a_t = g \Rightarrow 3gr\sin\theta/2h = g \Rightarrow \sin\theta = \frac{2h}{3r} \overset{r=h}{=\!=} \frac{2}{3} \Rightarrow \theta = 41.8°$

NOTE: NEWTON'S 2nd LAW FOR A BIT OF ROD dm @ r & θ

$$\vec{F}^{net}_{on\,dm} = (dm)\vec{a}_{of\,dm} = dm\left\{a_r\hat{r} + a_\theta\hat{\theta} = \frac{3gr}{h}(1-\cos\theta)(-\hat{r}) + \frac{3g}{2h}r\sin\theta\,\hat{\theta}\right\}$$

$$= (dm)\vec{g} + (d\vec{F} = \vec{F}(\text{from rod below } dm) - \vec{F}(\text{from rod above } dm))$$