STAEDTLER® No. 937 811E Engineer's Computation Pad

••13 A bowler throws a bowling ball of radius $R = 11$ cm along a lane. The ball (Fig. 11-37) slides on the lane with initial speed $v_{com,0} = 8.5$ m/s and initial angular speed $\omega_0 = 0$. The coefficient of kinetic friction between the ball and the lane is 0.21. The kinetic frictional force $\vec{f}_k$ acting on the ball causes a linear acceleration of the ball while producing a torque that causes an angular acceleration of the ball. When speed v_{com} has decreased enough and angular speed ω has increased enough, the ball stops sliding and then rolls smoothly. (a) What then is v_{com} in terms of ω? During the sliding, what are the ball's (b) linear acceleration and (c) angular acceleration? (d) How long does the ball slide? (e) How far does the ball slide? (f) What is the linear speed of the ball when smooth rolling begins?

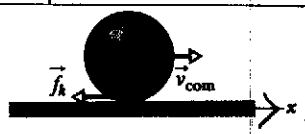


FIG. 11-37 Problem 13.

THE BALL SLIDES INITIALLY WHICH RESULTS IN A KINETIC FRICTION FORCE OF MAGNITUDE

$f_k = \mu_k N \xrightarrow[N=mg]{F_y^{net}=0} \mu_k mg$

ACTING IN THE DIRECTION TO OPPOSE RELATIVE MOTION OF THE SURFACES i.e. OPPOSITE $\vec{v}_{com}$.

$\vec{F}^{net}_{on\ BALL} = \frac{d\vec{P}_{ball}}{dt} = m_{ball}\vec{a}_{com\ of\ ball}$

$\mu_k mg(-\hat{x}) = m\vec{a}_{com}$

b) $\Rightarrow \vec{a} = \vec{a}_{com} = \mu_k g(-\hat{x}) \Rightarrow a = -\mu_k g = \text{CONSTANT}\ a = 2.1\ m/s^2$

$\Rightarrow v_{com} = v_{0,com} - \mu_k g t$; $x = (x_0=0) + v_{0,com}t - \frac{1}{2}\mu_k g t^2$; $v_{com}^2 = v_0^2 - 2\mu_k g x$

c) $\tau^{net}_{about\ com} = I_{com}\alpha_{c\ about\ com} \Rightarrow -f_k R = \left(\frac{2}{5}mR^2\right)\alpha_c$

$m\vec{g}$ & $\vec{N}$ have moment arms $\equiv 0$; $\vec{f}_k$ NOTE CLOCKWISE $\Rightarrow \tau < 0$, $\alpha < 0$

$\alpha_c = \frac{-5}{2}\frac{(f_k = \mu_k mg)}{mR} = -5\mu_k g/2R = \text{CONSTANT} = -46.7\ rad/s^2 \Rightarrow \omega_c = \omega_0 + \alpha_c t$

NOTE $\omega_0 \equiv 0$ & ω_c is about com

$\omega_c = -5\mu_k g t/2R$ INCREASES UNTILL THE RELATIONSHIP

a) $v'_{com} = R|\omega'_c|$ HOLDS THEN THE INSTANTANEOUS POINT OF CONTACT, P, OF THE BALL ON THE FLOOR IS AT REST AND ROLLING WITHOUT SLIPPING BEGINS. THE CONDITION OCCURS AT TIME $t = T$

$v_{com} = v'_{com}$ @ CRITICAL MOMENT = T

$\left(v'_{com} = v_0 - \mu_k g T\right) = R\left[|\omega'_c| = 5\mu_k g T/2R\right]$

d) $\Rightarrow v_0 = \mu_k g T\left[1 + 5/2\right] = 7\mu_k g T/2$

$\Rightarrow T = 2v_0/[7\mu_k g] = 1.18\ s$

e) $\Rightarrow v'_{com} = |Rw'_c| = R\frac{5\mu_k g}{2R}\frac{2v_0}{7\mu_k g} = \frac{5v_0}{7} = 6.07\ m/s$ @ $t = T$

e) DISTANCE BALL SLIDES $= x = \frac{v'^2_{com} - v_0^2}{2\mu_k g}$ OR $= v_0 T - \frac{1}{2}\mu_k g T^2$

$x = \left[(5/7)^2 - 1\right]v_0^2/2\mu_k g = \frac{12 v_0^2}{49\mu_k g} = 8.6\ m$

A SCALED DIAGRAM OF THE BALL ANGULARLY ACCELERATING (AS A RESULT OF THE FRICTION TORQUE) UP TO ANGULAR VELOCITY $\omega'_c = v'_{com}/R$. THIS OCCURED WHILE v_{com} DECREASED FROM $v_{com} = v_{com,0} = v_0$ TO $v_{com} = v'_{com}$ AS A RESULT friction DECELERATION

