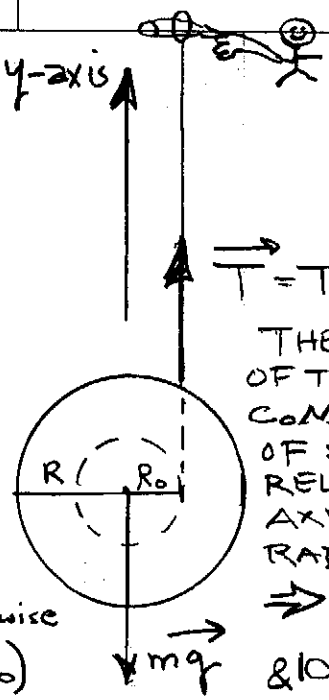
STAEDTLER® No. 937 811E Engineer's Computation Pad

sec. 11-5 The Yo-Yo

•17 A yo-yo has a rotational inertia of 950 g·cm² and a mass of 120 g. Its axle radius is 3.2 mm, and its string is 120 cm long. The yo-yo rolls from rest down to the end of the string. (a) What is the magnitude of its linear acceleration? (b) How long does it take to reach the end of the string? As it reaches the end of the string, what are its (c) linear speed, (d) translational kinetic energy, (e) rotational kinetic energy, and (f) angular speed? SSM



THE VELOCITY OF THE YO-YO CoM = VELOCITY OF STRING RELATIVE TO AXLE (WITH RADIUS = R_0)
$\Rightarrow |\omega_{com}| = \frac{|v_{com}|}{R_0}$ & $|\alpha_{com}| = \frac{|a_y|}{R_0}$

$\vec{F}_{net} = \frac{d\vec{p}}{dt} = m\vec{a} = ma_y = ma$

$\Rightarrow T - mg = ma \Rightarrow T = ma + mg$

NOTE: $mg > T \Rightarrow a < 0$

$\vec{\tau}_{net} = \frac{d\vec{L}}{dt} \xrightarrow{\text{about CoM}} \tau_{net} = I_{com}\alpha_{com}$

$\tau^{net\ ext}_{com} = \tau_{Gravity} + \tau_{Tension} = 0 + (+TR_0)$ (moment arm = 0; counterclockwise)

$\Rightarrow +TR_0 = I_{com}(\alpha = -a/R_0)$ since $a = a_y < 0$ & $\alpha > 0$

$\Rightarrow T = -I_{com}a/R_0^2$

$T = T \Rightarrow ma + mg = -I_{com}a/R_0^2 \Rightarrow a\left(m + \frac{I_{com}}{R_0^2}\right) = -mg$

a) $\Rightarrow a = \frac{-g}{(1 + I_{com}/mR_0^2)} = \frac{-980\ cm/s^2}{1 + 950/(120 * 0.32^2)} = -12.5\ cm/s^2 = -0.125\ m/s^2$

b) $a = a_{com} = a_y = \text{CONSTANT} \Rightarrow y = y_0 + v^{\overset{0}{}}_{0y}t + \frac{1}{2}at^2 \Rightarrow$ LENGTH OF STRING $= h = \frac{1}{2}|a|t^2 \Rightarrow t = \sqrt{2h/|a|} = \sqrt{2(120)/12.5} = 4.4\ s$

c) $v_{com} = v_0 + a_y t \Rightarrow v_{com} = 0 + -12.5\ \frac{cm}{s}\ 4.4\ s = 55\ \frac{cm}{s} = 0.55\ \frac{m}{s}$

d) $K_{TRANS} = \frac{1}{2}mv_{com}^2 = \frac{1}{2}(120\ g)\,55^2 = 1.8 \times 10^5\ ergs\left(\frac{joule}{10^7\ erg}\right) = 0.018\ Joules$

e) $K_{rot} = \frac{1}{2}I_{com}\omega_{com}^2 = \frac{1}{2}(950)\,172^2 = 1.4 \times 10^7\ erg\left(\frac{joule}{10^7\ erg}\right) = 1.4\ Joules!$

f) $\omega_{com} = v_{com}/R_0 = 55\ cm/s\ /\ 0.32\ cm = 172\ rad/s$
$= 172\ \frac{rad}{s}\ \frac{rev}{2\pi\ Rad} = 27\ rev/s$

CONSERVATION OF ENERGY FOR THE YO-YO: $\Delta E_{mech} \equiv 0$

$\Rightarrow E_i = E_f \Rightarrow K_i + U_i = K_f + U_f \Rightarrow K_i^{TRANS} + K_i^{rot} + U_i = K_f^T + K_f^R + U_f$

$\Rightarrow \frac{1}{2}mv_0^2 + \frac{1}{2}I_c\omega_0^2 + mg(y_i = H) = \frac{1}{2}mv_{com}^2 + \frac{1}{2}I_c\omega_{com}^2 + mgy$

$\frac{1}{2}\left(m + \frac{I_c}{R_0^2}\right)v_0^2 + mg(H - y) = \frac{1}{2}\left(m + \frac{I_c}{R_0^2}\right)v_{com}^2$

I ALLOW FOR PROJECTING THE YO-YO AT INITIAL $v_{com} = v_0$

$I_{com} = 2I_{c,\ DISK\ OF\ RADII = R} + I_{c,\ AXLE\ OF\ RADII = R_0} = 2\left(\frac{1}{2}M_D R^2\right) + \frac{1}{2}m_a R_0^2$

USUALLY $R_0 \ll R$ & $m_a \ll M_{DISK}$ SO $\frac{1}{2}m_a R_0^2 \approx 0$ RELATIVE TO $M_D R^2$