STAEDTLER® No. 937 811E Engineer's Computation Pad

•52 A bola consists of three massive, identical spheres connected to a common point by identical lengths of sturdy string (Fig. 11-51*a*). To launch this native South American weapon, you hold one of the spheres overhead and then rotate that hand about its wrist so as to rotate the other two spheres in a horizontal path about the hand. Once you manage sufficient rotation, you cast the weapon at a target. Initially the bola rotates around the previously held sphere at angular speed ω_i but then quickly changes so that the spheres rotate around the common connection point at angular speed ω_f (Fig. 11-51*b*). (a) What is the ratio ω_f/ω_i? (b) In the center-of-mass frame, what is the ratio K_f/K_i of the corresponding rotational kinetic energies?

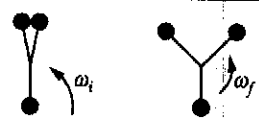


FIG. 11-51 Problem 52.

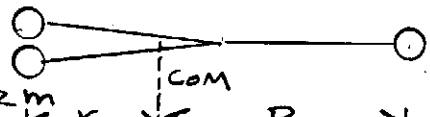
WHEN RELEASED THE COM FOLLOWS THE TRAJECTORY OF A PARABOLA ON EARTH & A STRAIGHT LINE WITH UNIFORM MOTION IN FORCE-FREE SPACE: $\vec{F}^{net}_{ext} = \frac{d\vec{P}_{total}}{dt} = \frac{dM_{tot}\vec{v}_{com}}{dt}$

or $\vec{F}^{net}_{ext} = M_{tot}\vec{a}_{com}$

IN THE COM FRAME OF REFERENCE WE HAVE $\vec{\tau}^{net'}_{ext} = \frac{d\vec{L}'_{total}}{dt}$ and $\vec{\tau}^{net'}_{ext} = \vec{\tau}^{com}_{gravity} = 0$ since the GRAVITATIONAL FORCE ACTS AT THE COM SO THE MOMENT ARM = 0

THEREFORE: $\vec{L}'_{total} = \text{CONSTANT} = \text{TOTAL ANG MOM ABOUT COM}$

THE POSITION OF THE COM IS SUCH THAT



INITIALLY $\sum Mr = mR \Rightarrow 2mr = mR$

$\Rightarrow R = 2r$ & $r+R = 2l \Rightarrow r = 2l - R$

$\Rightarrow R = 4l - 2R \Rightarrow R = 4l/3 \Rightarrow r = 2l/3$; $r+R = 2l$ (l = LENGTH OF A STRING)

$I^i_{com} = \sum_{i=1}^{2} m_i r_i^2 = mR^2 + (2m)r^2 = m\frac{16l^2}{9} + 2m\frac{4l^2}{9} = m\frac{24l^2}{9} = \frac{8}{3}ml^2$

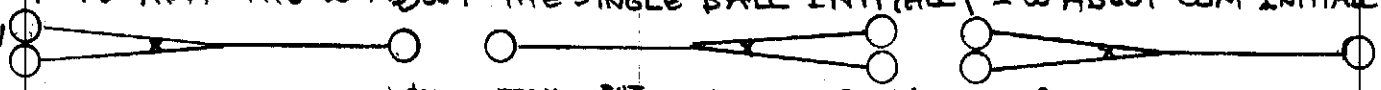
FINALLY $I^f_{com} = \sum_{i=1}^{3} m_i r_i^2 = 3ml^2$; NOTE $I^f_{com} > I^i_{com}$ SINCE $3 = \frac{9}{3}$

CONSERVATION OF ANGULAR MOMENTUM ABOUT THE COM: $\vec{L}^i_{com} = \vec{L}^f_{com}$

$\Rightarrow I^i_{com}\omega^{com}_i = I^f_{com}\omega^{com}_f \Rightarrow \frac{\omega_f}{\omega_i} = \frac{I^i_{com}}{I^f_{com}} = \frac{8}{9} < 1$ Like a ROTATING ICE SKATER OPENING HER ARM, THE BOLA SLOWS ITS ROTATION!

NOTE THAT THE ω ABOUT THE SINGLE BALL INITIALLY = ω ABOUT COM INITIALLY

ONE ROTATION



IN THE LAB FRAME: $K^{total} = K^{TRAN} + K^{ROT}_{com} = \frac{1}{2}M_{tot}v^2_{com} + \frac{1}{2}I_{com}\omega^2_{com}$

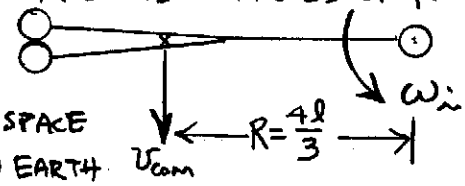
IN THE LAB FRAME: $\vec{L}_{total} = \vec{r}_{com} \times (\vec{P}_{tot} = M_{tot}\vec{v}_{com}) + (\vec{L}'_{com} = I_{com}\vec{\omega}_{com})$

$\frac{K'^{rot}_{com,f}}{K'^{rot}_{com,i}} = \frac{\frac{1}{2}I^f_{com}\omega'^2_f}{\frac{1}{2}I^i_{com}\omega'^2_i} = \frac{L^f_{com}\omega'_f}{L^i_{com}\omega'_i} = \frac{\omega'_f}{\omega'_i} = \frac{\omega^{com}_f}{\omega^{com}_i} = \frac{8}{9} < 1$ (K IS LOST IN THE RECONFIGURATION OF THE BALLS!)

THE ICE SKATER FEELS GAIN & LOSS AS SHE REPEATEDLY OPENS & CLOSES HER ARMS!
CENTRIPETAL FORCES POINT INWARD. DISPLACEMENT: OUTWARD $\Rightarrow W<0$ & INWARD $\Rightarrow W>0$
INITIALLY THE ROTATION IN THE INERTIAL FRAME (LAB) IS ABOUT THE WRIST OF THE SINGLE BALL WHICH DOES NOT COINCIDE WITH THE COM, THE...
... ROTATION IS NOT STABLE WHEN THE CENTER OF MASS & CENTER OF ROTATION DO NOT COINCIDE. THE BOLA IS NOT A RIGID BODY AND THE TENSIONS IN THE STRINGS (WHICH OBEY NEWTON'S 3RD LAW) ACT TO RECONFIGURE THE MASS DISTRIBUTION SO THAT THE CENTER OF ROTATION AND THE CENTER OF MASS COINSIDE. THE ACTION-REACTION INTERNAL TORQUES THEY EXERT CANCEL OUT SO THAT THE ANGULAR MOMENTUM ABOUT THE COM DOES NOT CHANGE. THE TENSION FORCES, HOWEVER DO NEGATIVE INTERNAL WORK IN RECONFIGURING THE MASSES WHICH RESULTS IN A LOSS OF KINETIK ENERGY

EXTRA: THE INITIAL VELOCITY OF THE COM

$v^{com}_{initial} = R\omega_i = \frac{4}{3}l\omega_i$



$\vec{v}_{com}$ REMAINS CONSTANT IN FORCE-FREE SPACE

THE HORIZONTAL COMPONENT IS CONSTANT ON EARTH