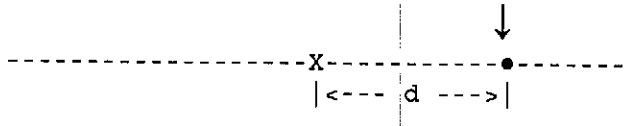
P.11.63

The Figure shows an overhead view of a thin uniform rod of length L = 0.600 m and mass M rotating horizontally at $\omega_{initial}$ = 80.0 rad/s counterclockwise about an axis through its center. A particle of mass m = M/3.00 and traveling horizontally at speed v = 40.0 m/s hits the rod and sticks. The particle's path is perpendicular to the rod at the instant of the hit, at a distance d from the rod's center.



A view from above

a) At what value of d are the rod and particle stationary after the hit?
b) In which direction does the rod and particle rotate if d is greater than this value?

If there are no External Torques acting on the System of "Rotating Rod + Particle", as Calculated about the "Rotation Axis";
Then The Total Angular Momentum of that System about that Axis = Constant; ie ...

$\tau_{net,ext} = dL_{Total}/dt = 0$ => The Total Angular Momentum about that Axis = L_{Total} = Constant

$L_{Total} = L_{rod} + \ell$

where initially for the Single Particle: $\ell = \mathbf{r} \times \mathbf{p}$ = (Moment arm = d)(p = mv)sin90°**in**
and initially for the Rod: $L_{rod} = I_{Rod}\omega_{initial} = (ML^2/12)(\omega)$**out**
and finally for the System: $L = I_{System}\omega_{final} = [(ML^2/12)+md^2]\omega_{final}$

$L_{Total_Initial} = L_{Total_Final}$

$\{(ML^2/12)(\omega_{initial}) - dmv\}$**out** $= [(ML^2/12) + md^2]\omega_{final}$

$\omega_{final} = \{(ML^2/12)(\omega_{initial}) - dmv\}/[(ML^2/12)+md^2]$**out**

$\omega_{final} = 0 \Rightarrow (ML^2/12)(\omega_{initial}) = dmv$

a) $d = (ML^2/12)(\omega_{initial})/(mv) = (M0.600^2/12)80.0/[(M/3.00)40.0] = 0.36/2 = 0.180$ m
b) $\omega_{final} = \{(ML^2/12)(\omega_{initial}) - dmv\}/[(ML^2/12)+md^2]$**out** = (-#)**out** = (#)**in** => Clockwise

NOTE: Create a new Problem with the orientation of this System being Vertical:
If this System was Vertical at the instant of the collision, and the rotation of the rod was instantaneously equal to $\omega_{initial}$ = 80.0 rad/s counterclockwise about an axis through its center, and the Particle was traveling horizontally at speed v = 40.0 m/s at the instant of the Collision, ...
then the result would be the same at the moment of the collision; however the External Torque of Gravity would change the Angular Momentum after the Collision.
Reason: The Torque due to gravity about the Rotation Axis on the System when it is in the Vertical orientation is Zero! You should be able to show this!
Therefore the Angular Momentum of the System is conserved for this instant.
This Principle is in effect for your Homework Problems: P.11.61 & P.11.66.