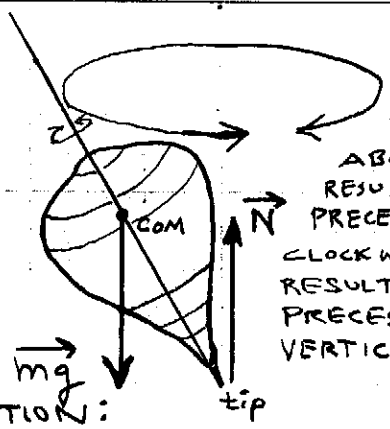
sec. 11-12 Precession of a Gyroscope

••68 A top spins at 30 rev/s about an axis that makes an angle of 30° with the vertical. The mass of the top is 0.50 kg, its rotational inertia about its central axis is 5.0×10^{-4} kg · m², and its center of mass is 4.0 cm from the pivot point.

If the spin is clockwise from an overhead view, what are the (a) precession rate and (b) direction of the precession as viewed from overhead?



SEEN FROM OVERHEAD COUNTERCLOCKWISE TOP ROTATION (IE ABOUT SYMMETRY AXIS) RESULTS IN COUNTERCLOCKWISE PRECESSION (ABOUT VERTICAL AXIS)

CLOCKWISE TOP ROTATION RESULTS IN CLOCKWISE PRECESSION ABOUT THE VERTICAL AXIS.

$\vec{\tau}_{ext}^{net} = \frac{d\vec{L}}{dt}$ $\vec{L}$ = ANGULAR MOMENTUM OF TOP

THE PHENOMENA OR OBSERVED MOTION:

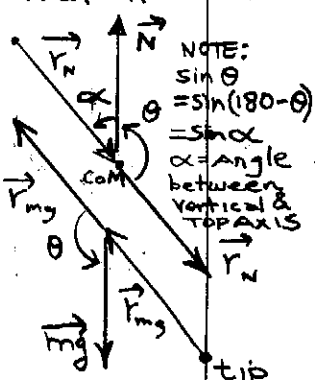
THE TOP IS SPINNING FURIOUSLY ABOUT ITS SYMMETRY AXIS: $\vec{\omega} = \omega_s \hat{\text{Axis}}$ WHILE THE TOP'S SYMMETRY AXIS OR SPIN AXIS SLOWLY PRECESSES (SLOW "CONING MOTION") ABOUT A VERTICAL AXIS PASSING THROUGH THE TOP'S TIP. $\Rightarrow \vec{L}_{\text{total IN LAB}} = \vec{L}^{\text{ABOUT SYMMETRY AXIS}}_{\text{ABOUT COM AXIS}} + \vec{L}^{\text{ABOUT VERTICAL AXIS}}_{\text{IN LAB}} \approx \vec{L}^{\text{TOP'S AXIS}}_{\text{COM \& LAB}}$ IE. IGNORE PRECESSION CONTRIBUTION.

THE STABLE TRANSLATIONAL STATE OF THE TOP $\Rightarrow m\vec{g} = -\vec{N} \Rightarrow N = mg$

TWO LOGICAL ORIGINS TO COMPUTE TORQUES ABOUT TO APPLY ✱ ARE:

TOP'S COM: $\vec{\tau}_{net} = \vec{\tau}_{m\vec{g}} + \vec{\tau}_{\vec{N}} = 0 + \vec{r}_N \times \vec{N} = \ell mg \sin\theta \,\hat{\text{out}}$ (ℓ = distance from tip to the COM)

TOP'S TIP: $\vec{\tau}_{net} = \vec{\tau}_{\vec{N}} + \vec{\tau}_{m\vec{g}} = 0 + \vec{r}_{mg} \times m\vec{g} = \ell mg \sin\theta \,\hat{\text{out}}$



NOTE: $\sin\theta = \sin(180-\theta) = \sin\alpha$; α = Angle between Vertical & Top Axis

THE RESULTS ARE THE SAME — THE SAME CHANGE RESULTS FOR THE ANGULAR MOMENTUM VECTOR $\vec{L}$

$\Delta\vec{L} = \vec{L}_f - \vec{L}_i \rightarrow d\vec{L} = \vec{L}_f - \vec{L}_i$ & $d\vec{L} = \vec{\tau}_{ext}^{net}\, dt$

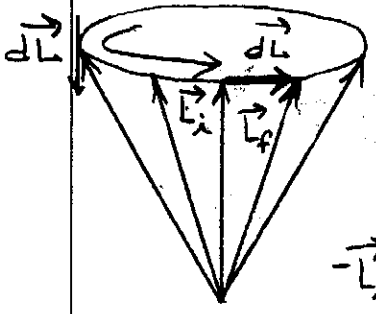
$\vec{L}_{top} = \vec{L}_{com} = \vec{L}_c = I_{com}\vec{\omega}_{\text{TOP AXIS}} = I_c \omega \,\hat{\text{RHR}}$ WITH 2 POSSIBILITIES

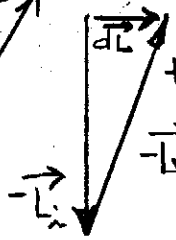
FOR DIRECTION DEPENDING ON SENSE OF ROTATION:

COUNTERCLOCKWISE → TOP ROTATION & PRECESSION ← CLOCKWISE

$d\vec{L} = \vec{\tau}_{ext}^{net}\, dt$

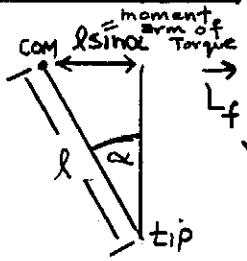
SEE THIS "CHANGE IN $\vec{L}$" AS:

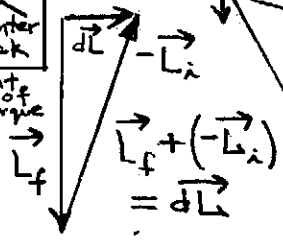


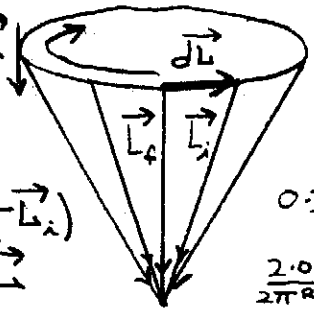


$\boxed{dL = mg\ell \sin\alpha\, dt}$ counter clock

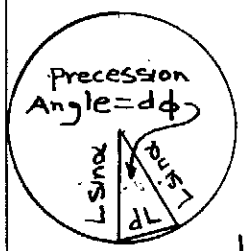
$-\vec{L}_i + \vec{L}_f = d\vec{L}$







0.33 $\frac{\text{REV}}{\text{SEC}}$ = 2.08 $\frac{\text{RAD}}{\text{SEC}}$ $\frac{2\pi\ \text{RAD}}{\text{REV}}$ = 2.08 $\frac{\text{RAD}}{\text{SEC}}$



$d\phi = \frac{dL}{L\sin\alpha} = \frac{mg\ell\sin\alpha\, dt}{L\sin\alpha} \Rightarrow \Omega \equiv \frac{d\phi}{dt} = \frac{mg\ell}{L} = \frac{mg\ell}{I_c\omega} = \frac{(0.5)(9.8)(0.04)}{5\times10^{-4}\,30(2\pi)}$

PRECESSION ANGULAR VELOCITY

ω = TOP'S ANG VEL = $\frac{30(2\pi)}{\text{RAD/S}}$

NOTE: $\Omega \propto \frac{1}{\omega} \Rightarrow$ THE GREATER ω_{spin}^{tops}, THE SLOWER $\Omega_{\text{RATE}}^{\text{Precession}}$.

ALSO KEEP IN MIND ENERGY CONSERVATION WHEN SETTING A TOP IN MOTION: SPIN THE TOP ABOUT A VERTICAL AXIS SO $\vec{L}$ IS VERTICAL AND THERE IS NO PRECESSION. GIVE THE AXIS A SIDEWAYS IMPULSE TO TIP IT SOMEWHAT — IT WILL BEGIN TO PRECESS IN ORDER TO CONSERVE THE VERTICAL COMPONENT OF $\vec{L}$ — THE KINETIC ENERGY FOR THE PRECESSIONAL MOTION COMES FROM THE LOSS OF POTENTIAL ENERGY WHEN THE COM OF THE TOP LOWERS AS THE TOP TIPS.

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$\vec{L}_{total}$ about COM = $\vec{r}_{COM} \times \vec{p}_{total}$ + $\vec{L}_{COM}$ IN LAB