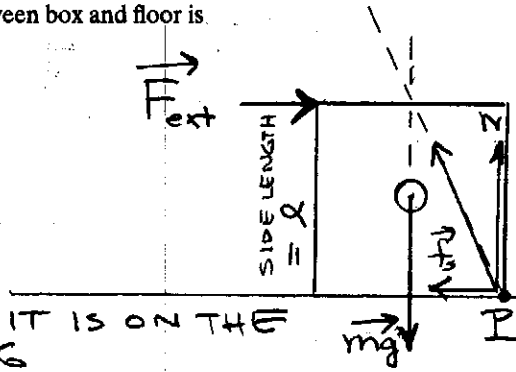
••33 A cubical box is filled with sand and weighs 890 N. We wish to "roll" the box by pushing horizontally on one of the upper edges. (a) What minimum force is required? (b) What minimum coefficient of static friction between box and floor is required? (c) If there is a more efficient way to roll the box, find the smallest possible force that would have to be applied directly to the box to roll it. (*Hint:* At the onset of tipping, where is the normal force located?)



ONLY WHEN THE BOX IS ON THE VERGE OF TIPPING CAN WE SAY EXACTLY WHERE THE NORMAL FORCE ACTS - IT ACTS AT THE POINT P ABOUT WHICH IT WILL TIP. THIS IS WHERE f_{static} ACTS ALSO

THE BOX IS IN EQUILIBRIUM WHEN IT IS ON THE VERGE OF TIPPING

$\Rightarrow \vec{F}^{net}_{on\ box} = 0 \Rightarrow \vec{F}_{ext} + m\vec{g} + \vec{F}^{floor}_{on\ box}$ & $\vec{F}^{floor}_{on\ box} = \vec{N} + \vec{f}_{static}$

$F^{net}_{vertical} = 0 \Rightarrow N = mg$ & $F^{net}_{horizontal} = 0 \Rightarrow F_{ext} - f_s = 0 \Rightarrow f_s = F_{ext}$

$\tau^{net}_{about\ P} = 0 \Rightarrow \tau_{F_{ext}} + \tau_{mg} + \tau_N + \tau_{f_s} = -F_{ext}\,\ell + mg\frac{\ell}{2} + 0 + 0 = 0$ (ℓ = SIDE LENGTH)

Since the Lines of Action of $\vec{N}$ & $\vec{f}_s$ pass through P $\Rightarrow$ their Moment Arms = 0. Also $\tau_{gravity} > 0$ (counterclockwise) & $\tau_{F_{ext}} < 0$.

a) $\Rightarrow F_{ext} = mg/2$ For Cubical Box on VERGE OF TIPPING.

b) $\Rightarrow f_{static} \le \mu_s N = \mu_s mg$ but $f_s = F_{ext} = mg/2$

$\Rightarrow$ THE MINIMUM $\mu_s = 1/2$ THIS μ_s BARELY ALLOWS $f_s = F_{ext}$

NOTE THE LAW: WHEN THERE ARE ONLY 3 FORCES THAT ACT ON A BODY IN A PLANE, AND THESE FORCES HAVE IDENTIFIABLE LOCATIONS AND THEREFORE LINES OF ACTION, AND 2 OF THE FORCES HAVE "LINES OF ACTION" WHICH INTERSECT, THEN THE 3RD FORCE'S LINE OF ACTION ALSO INTERSECTS THE POINT OF INTERSECTION OF THE OTHER 2 FORCE'S LINES OF ACTION - IF THE BODY IS IN EQUILIBRIUM. BECAUSE THE NET TORQUE ABOUT AN ARBITRARY POINT IN THE PLANE IS 0 - PICK THE PT WHERE THE 1ST 2 LINES OF ACTION INTERSECT - THEN THE 3RD MUST INTERSECT ALSO OR IT WOULD YIELD A NONZERO TORQUE ABOUT THIS POINT. (SEE THE DIAGRAM DRAWN TO SCALE ABOVE!)

FOR A BODY IN EQUILIBRIUM UNDER THE ACTION OF 3 FORCES IN A PLANE

THE SMALLEST POSSIBLE FORCE THAT GIVES THE REQUIRED TORQUE TO TIP IS THE FORCE WITH THE LARGEST MOMENT ARM WHICH WOULD BE THE DIAGONAL OF A SIDE = $\ell\sqrt{2}$: $\tau^{net}_{required} = 0 = -F_{ext}(\sqrt{2}\,\ell) + mg\,\ell/2 = 0$

$\Rightarrow F_{ext} = mg/(2\sqrt{2}) = 0.354\,mg$. We can do this by brute force:

$F^{net}_{vert} = 0 \Rightarrow F^{ext}\sin\theta + N - mg = 0$

$F^{net}_{horiz} = 0 \Rightarrow F_{ext}\cos\theta - f_s = 0$

$\tau^{net} = 0 \Rightarrow -F_{ext}\cos\theta(\ell) + {}^{-}F_{ext}\sin\theta(\ell) + {}^{+}mg(\ell/2) = 0$

$\Rightarrow F_{ext} = mg/[2(\cos\theta + \sin\theta)]$ = MAX WHEN...

$\cos\theta + \sin\theta$ = min $\Rightarrow \frac{d}{d\theta}(\cos\theta + \sin\theta) = 0$

$\Rightarrow -\sin\theta + \cos\theta = 0 \Rightarrow \sin\theta = \cos\theta \Rightarrow \theta = 45°$

$\Rightarrow F_{ext} = mg/\left[2\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)\right] = mg/(2\sqrt{2})$ ✓

$\Rightarrow N = mg - \frac{1}{\sqrt{2}}(mg/2\sqrt{2}) = 3mg/4$ & $f_s = (mg/2\sqrt{2})\left(\frac{1}{\sqrt{2}}\right) = \frac{mg}{4}$

