STAEDTLER® No. 937 811E Engineer's Computation Pad

59 Four bricks of length L, identical and uniform, are stacked on top of one another (Fig. 12-69) in such a way that part of each extends beyond the one beneath. Find, in terms of L, the maximum values of (a) a_1, (b) a_2, (c) a_3, (d) a_4, and (e) h, such that the stack is in equilibrium.

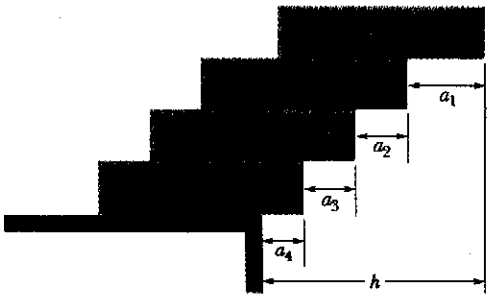
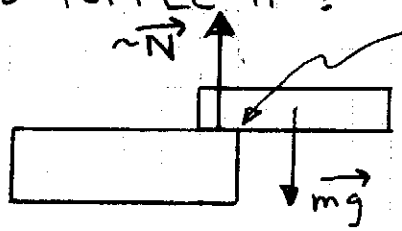
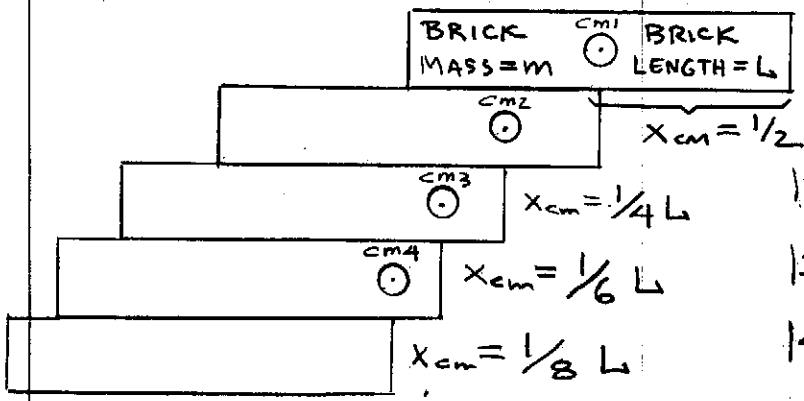


FIG. 12-69 Problem 59.

THE CENTER OF MASS OF THE OVERLYING WEIGHT MUST OVERLAY THE BASE THAT SUPPORTS IT - OTHERWISE BOTH THE WEIGHT & THE NORMAL FORCE WOULD EXERT TORQUES TO TOPPLE IT!





$x_{cm} = \frac{1}{2n} L$

X-axis: X=0 = R-EDGE OF BLOCK ON TOP

$x_{cm} = (m_1 x_1 + m_2 x_2)/(m_1 + m_2)$

|1⟩ $x_{cm} = [0(x_1=0) + m(L/2)]/m$

|2⟩ $x_{cm} = [m(x_1=0) + m(L/2)]/2m$

|3⟩ $x_{cm} = [2m(x_1=0) + m(L/2)]/3m$

|4⟩ $x_{cm} = [3m(x_1=0) + m(L/2)]/4m$

⋮

|n⟩ $x_{cm} = [(n-1)m(x_1=0) + m(L/2)]/nm$

$$h_{\text{4 bricks}} = \frac{1}{2} L \left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right)$$

THIS IS REALLY 5 BRICKS UNLESS THE 4th is OVERLAYING A TABLE,

$$= \frac{1}{2} L (2.08) = 1.04 L$$

NOTE: MY DIAGRAM IS TO SCALE!

THE SERIES: $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$ IS THE HARMONIC SERIES NAMED FROM THE CONCEPT OF OVERTONES OR HARMONICS IN MUSIC THE WAVELENGTHS OF THE OVERTONES OF A VIBRATING STRING ARE: $1/2, 1/3, 1/4, \cdots$ OF THE STRINGS FUNDAMENTAL WAVELENGTH EVERY TERM OF THE SERIES AFTER THE 1ST IS THE HARMONIC MEAN OF THE PRECEEDING & FOLLOWING TERMS (TERM = HARMONIC MEAN DERIVES FROM MUSIC). THE SERIES SURPRISINGLY DIVERGES (PROOF FOLLOWS):

$$\text{SUM} = 1 + \left[\frac{1}{2}\right] + \left[\frac{1}{3} + \frac{1}{4}\right] + \left[\frac{1}{5} + \frac{1}{6} + \frac{1}{17} + \frac{1}{8}\right] + \left[\frac{1}{9} + \cdots + \frac{1}{16}\right] + \cdots$$

$$> 1 + \left[\frac{1}{2}\right] + \left[\frac{1}{4} + \frac{1}{4}\right] + \left[\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}\right] + \left[8 \times \frac{1}{16}\right] + \cdots$$

$$= 1 + \left[\frac{1}{2}\right] + \left[\frac{1}{2}\right] + \left[\frac{1}{2}\right] + \left[\frac{1}{2}\right] + \cdots$$

$$= \infty$$

IT IS AMAZING that the first 10^{43} terms SUM < 100