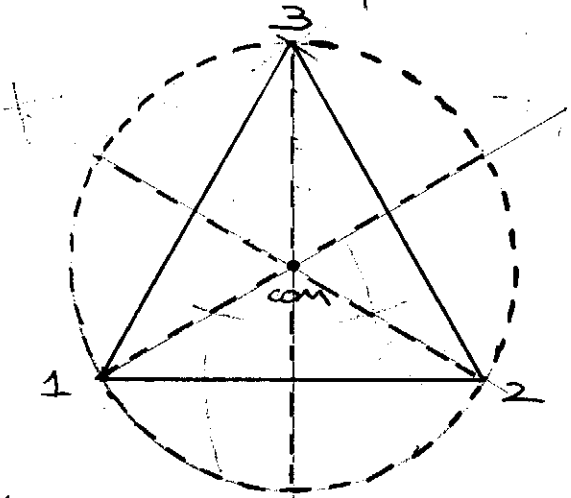
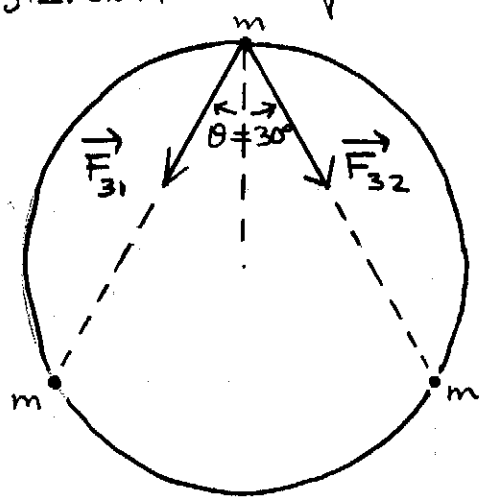
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•••59 Three identical stars of mass M form an equilateral triangle that rotates around the triangle's center as the stars move in a common circle about that center. The triangle has edge length L. What is the speed of the stars?

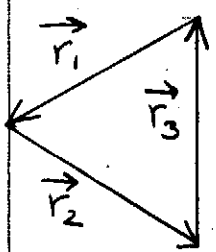
THE 3 STARS ORBIT THEIR COMMON CENTER OF MASS... WHICH HAPPENS TO BE THE TRIANGLE'S CENTER BY SYMMETRY. EACH STAR OBEYS NEWTON'S 2ND LAW FOR UNIFORM CIRCULAR MOTION:

$$\vec{F}^{\text{net}}_{\text{on m}} = m\vec{a}_{\text{centripetal}} \Rightarrow F^{\text{net}}_{\text{grav on m}} = m\frac{v^2}{r}$$

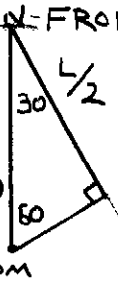




THE "CENTER" OF THE TRIANGLE FOUND BY DRAWING ⊥-BISECTORS TO EACH SIDE OR OPPOSITE ANGLE AND FINDING THE COMMON POINT OF INTERSECTION IS CLEARLY THE COM BECAUSE $m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 = m(\vec{r}_1 + \vec{r}_2 + \vec{r}_3) \equiv 0$ WHERE $\vec{r}_c$ IS DRAWN FROM THE COM



IT'S LENGTH IS OBTAINED BY: $\frac{L/2}{r_c} = \cos 30$



$$\Rightarrow r_c = \frac{L/2}{\sqrt{3}/2} = L/\sqrt{3}$$

BY THE PRINCIPLE OF LINEAR SUPERPOSITION THE NET GRAVITATIONAL FORCE ON ONE OF THE MASSES FROM THE OTHER TWO IS

$$\text{eg } \vec{F}^{\text{net}}_{\text{on 3}} = \vec{F}_{31} + \vec{F}_{32}$$

NOTE THAT THE TANGENTIAL COMPONENTS (TO THE CIRCLE) OF THE 2 FORCES SUM TO 0 BY SYMMETRY. THE RADIAL COMPONENTS ARE EQUAL & ADD TO

$$\vec{F}^{\text{net}}_{\text{on m}} = 2\left(F = \frac{Gmm}{L^2}\right)\cos 30^\circ$$

$$= \sqrt{3}\,Gm^2/L^2 \text{ to center}$$

$$\vec{F}^{\text{net}}_{\substack{\text{GRAVITY}\\\text{on m}}} = \vec{F}_{\substack{\text{centrip}\\\text{etal}}} = ma_c \Rightarrow \frac{\sqrt{3}\,Gm^2}{L^2} = m\frac{v^2}{(r_c = L/\sqrt{3})}$$

$$\Rightarrow v = \sqrt{\frac{Gm}{L}}$$

