STAEDTLER® No. 937 811E Engineer's Computation Pad

99 Several planets (Jupiter, Saturn, Uranus) are encircled by rings, perhaps composed of material that failed to form a satellite. In addition, many galaxies contain ring-like structures. Consider a homogeneous thin ring of mass M and outer radius R (Fig. 13-55). (a) What gravitational attraction does it exert on a particle of mass m located on the ring's central axis a distance x from the ring center? (b) Suppose the particle falls from rest as a result of the attraction of the ring of matter. What is the speed with which it passes through the center of the ring?

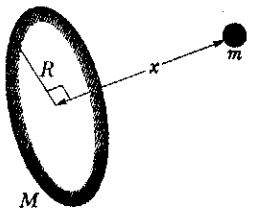
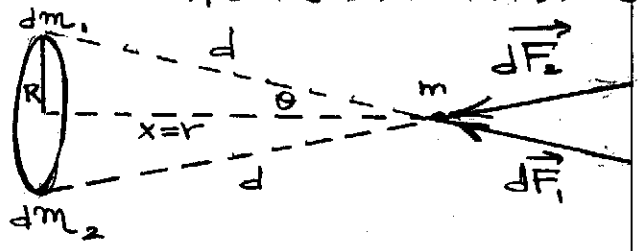


FIG. 13-55 Problem 99.

FIND THE FORCE ON m DUE TO AN INFINITESIMAL BIT, dm OF THE RING AND NOTE THE CONTRIBUTION OF IT'S PARTNER ON THE OPPOSITE SIDE OF A DIAMETER



THE FORCE DUE TO $dm = \vec{dF} = G\frac{m\,dm}{d^2}$ toward dm

THE GRAVITATIONAL POTENTIAL ENERGY due to $dm = dU = -\frac{Gm\,dm}{d}$

ONLY THE AXIAL COMPONENTS OF $\vec{dF}$ ADD UP TO $\vec{F}_{\text{total due to RING}}$ THE $\perp$-COMPONENTS SUM TO 0 BY SYMMETRY

$$\vec{F}_{\substack{\text{on } m \\ \text{by RING}}} = \int dF_{\text{AXIAL}} = \int \frac{Gm}{d^2}\left[\cos\theta = \frac{r}{d}\right] dm$$ BY PRINCIPLE OF LINEAR SUPERPOSITION

FORCE HAS DIRECTION $-\hat{r}$ or $-\hat{x}$

$$= \left[\frac{Gmr}{d^3} = \frac{Gmr}{\left(\sqrt{R^2+r^2}\right)^3}\right] \int dm = \frac{GmMr}{(R^2+r^2)^{3/2}}$$ WHERE M = mass of RING

$$U_{\text{total}} = \int dU = -\int \frac{Gm}{d}\,dm = \frac{-Gm}{\sqrt{r^2+R^2}} \int dm = \frac{-GmM}{\sqrt{r^2+R^2}}$$

CONSERVATION OF ENERGY: $E_{\text{initial}} = E_{\text{final}}$ or $\Delta E \equiv 0$

$$\frac{1}{2}mv_i^2 + \frac{-GmM}{\sqrt{r_i^2+R^2}} = \frac{1}{2}mv_f^2 + \frac{-GmM}{\sqrt{r_f^2+R^2}}$$

$$0 + \frac{-GmM}{\sqrt{x^2+R^2}} = \frac{1}{2}mv^2 + \frac{-GmM}{\sqrt{0^2+R^2}}$$

$$\Rightarrow v^2 = \frac{2GM}{R} - \frac{2GM}{\sqrt{x^2+R^2}}$$

WE CAN USE THE RESULT FOR THE GRAVITATIONAL FORCE DUE TO A RING OF MASS TO SOLVE (AS NEWTON DID) THE PROBLEM OF THE NET FORCE ON A PARTICLE OF MASS m DUE TO A SPHERICAL SHELL OF MASS, WHICH RESULTS IN 2 PROBLEMS:

1> POINT MASS m OUTSIDE OF A SHELL OF MASS

2> POINT MASS m INSIDE OF A SHELL OF MASS.