STAEDTLER® No. 937 811E Engineer's Computation Pad

•••**24** In Fig. 14-37, water stands at depth $D = 35.0$ m behind the vertical upstream face of a dam of width $W = 314$ m. Find (a) the net horizontal force on the dam from the gauge pressure of the water and (b) the net torque due to that force about a line through O parallel to the width of the dam. (c) Find the moment arm of this torque.

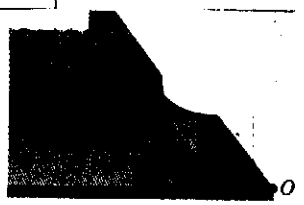
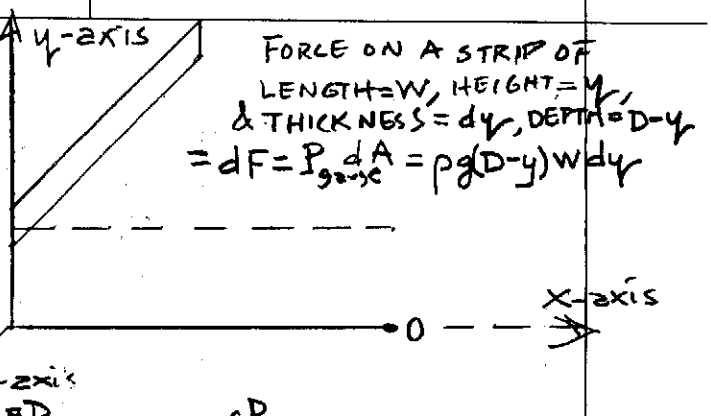


FIG. 14-37 Problem 24.



TOTAL FORCE ON DAM = $F = \int dF = \int_{y=0}^{y=D} P_{gauge}\, dA = \int_0^D \rho g (D-y) W\, dy$

$F = \rho g W \int_{y'=0}^{y'=D} y'(-dy')$ WHERE $y' = D-y \Rightarrow dy' = 0 - dy$ & $y=0 \Rightarrow y'=D$, $y=D \Rightarrow y'=0$

$F = \rho g W \int_0^D y'\, dy' = \rho g W \frac{y'^2}{2}\Big|_0^D = \rho g W D^2/2$ NOTE $F \propto D^2$!

a) $F = \left[\rho = \frac{1\ \text{METRIC TON} = 1000\ \text{kg}}{\text{CUBIC METER}}\right] 9.80 \frac{m}{s^2}\, 314\text{m}\, 35^2\text{m}^2/2 = 1.88 \times 10^9\ \text{N}$!

b) $d\tau$ = FORCE × MOMENT ARM $= dF(y) = \rho g (D-y) W y\, dy$

$\tau = \int d\tau = \int_0^D \rho g W (Dy - y^2)\, dy = \rho g W \left\{ D\frac{y^2}{2} - \frac{y^3}{3} \right\}_0^D$

$\tau = \rho g W \left[\frac{D^3}{2} - \frac{D^3}{3}\right] = \rho g W D^3 \left[\frac{3-2}{6}\right] = \rho g W D^3/6$

$\tau = 10^3 \frac{\text{kg}}{\text{m}^3}\, 9.80 \frac{m}{s^2} (314\text{m})\, 35^3\text{m}^3/6 = 2.20 \times 10^{10}\ \text{Nm}$

c) DEFINE THE EFFECTIVE MOMENT ARM VIA:

$\tau_{NET} = F_{NET}\, l_G \Rightarrow l_{EFFECTIVE} = \frac{\rho g W D^3/6}{\rho g W D^2/2} = \frac{D}{3}$

THE FORCE IS EFFECTIVELY APPLIED AT $\frac{2}{3}$RDS THE DEPTH