STAEDTLER® No. 937 811E Engineer's Computation Pad

••29 In Fig. 14-39, a spring of spring constant 3.00×10^4 N/m is between a rigid beam and the output piston of a hydraulic lever. An empty container with negligible mass sits on the input piston. The input piston has area A_i, and the output piston has area $18.0A_i$. Initially the spring is at its rest length. How many kilograms of sand must be (slowly) poured into the container to compress the spring by 5.00 cm?

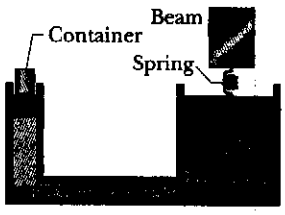


FIG. 14-39 Problem 29.

EQUILIBRIUM AT ANY GIVEN MOMENT (REASON FOR THE WORD "SLOWLY")

$$\Rightarrow \frac{F_{input}}{A_i} = \frac{F_{output}}{A_o}$$

PASCAL'S PRINCIPLE: THE IMPRESSED EXTERNAL PRESSURES... ... ARE TRANSMITTED UNDIMINISHED THROUGH THE INCOMPRESSIBLE FLUID (NOTE THE FLUID EXERTS EQUAL & OPPOSITE PRESSURES TO THAT IMPRESSED BY THE PISTONS BY NEWTON'S 3rd LAW, AND EQUILIBRIUM WITH A HYDROSTATIC FLUID REQUIRES THESE PRESSURES TO BE EQUAL AT OUTPUT & INPUT ENDS)

$$F_{OUTPUT} = \frac{A_o}{A_i}(F_{input} = mg)$$

WE WANT $F_{OUTPUT} = F_{SPRING} = k\Delta x_o$ WHERE $\Delta x_o = 5\text{cm}$

$$\Rightarrow F_{OUTPUT} = k\Delta x_o = \frac{A_o}{A_i} mg$$

$$\Rightarrow m = k\Delta x_o A_i / (A_o g) = 3.00\times10^4 (0.05) / (18 * 9.80) = 8.50\,\text{kg}$$

extra HOW FAR DOES THE INPUT PISTON MOVE?

SINCE THE FLUID IS INCOMPRESSIBLE $\Delta V_i = -\Delta V_o$

$$\Rightarrow \Delta x_i A_i = \Delta x_o A_o \Rightarrow \Delta x_i = \frac{A_o}{A_i}\Delta x_o = 18\Delta x_o$$

ENERGY IS CONSERVED $\Rightarrow W^{INPUT}_{FORCE} = \Delta U_g + \Delta U_s$