STAEDTLER® No. 937 811E Engineer's Computation Pad

•••30 Use the wave equation to find the speed of a wave given in terms of the general function $h(x, t)$:

$$y(x, t) = (4.00\text{ mm})\, h[(30\text{ m}^{-1})x + (6.0\text{ s}^{-1})t].$$

THE WAVE EQUATION

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

NOTE THAT DIMENSIONAL ANALYSIS VIA UNITS HELPS TO MEMORIZE THIS EQUATION

PHYSICALLY IT SAYS THAT THE CURVATURE OF THE WAVE FORM (2ND DERIVATIVE OF "y = WAVE FORM" WRT "DISTANCE = x" $= \frac{\partial^2 y}{\partial x^2}$ = RATE OF CHANGE OF SLOPE) IS PROPORTIONAL TO THE ACCELERATION (OF THE PARTICLES EXECUTING THE SHMOTION). THIS MAKE SENCE AND WE DERIVED IT FROM NEWTON'S LAWS.

SYMBOLICLY WRITE $y(x,t) = A\,h(kx \pm \omega t) = A\,h(\phi)$

HERE h = AN ARBITRARY FUNCTION THAT MAKES SENCE AS A WAVE FORM, AND
A = AN AMPLITUDE & $\phi = kx \mp \omega t$

1ST FIND THE LHS = LEFT-HAND SIDE USING THE CHAIN RULE

$$\frac{\partial y}{\partial x} = A\frac{\partial h}{\partial \phi}\frac{\partial \phi}{\partial x} = A\frac{\partial h}{\partial \phi}(k) = Ak\frac{\partial h}{\partial \phi}$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{\partial}{\partial x}\left\{Ak\frac{\partial h}{\partial \phi}\right\} = \frac{\partial}{\partial \phi}\left\{Ak\frac{\partial h}{\partial \phi}\right\}\frac{\partial \phi}{\partial x} = \left[Ak\frac{\partial^2 h}{\partial \phi^2}\right]k$$

$$= Ak^2\frac{\partial^2 h}{\partial \phi^2}$$

2ND FIND THE RHS = RIGHT HAND SIDE

$$\frac{\partial y}{\partial t} = \frac{\partial y}{\partial \phi}\frac{\partial \phi}{\partial t} = \left[A\frac{\partial h}{\partial \phi}\right](\mp\omega) = \mp A\omega\frac{\partial h}{\partial \phi}$$

$$\frac{\partial^2 y}{\partial t^2} = \frac{\partial}{\partial t}\left\{\mp A\omega\frac{\partial h}{\partial \phi}\right\} = \frac{\partial}{\partial \phi}\left\{\mp A\omega\frac{\partial h}{\partial \phi}\right\}\frac{\partial \phi}{\partial t}$$

$$= \left\{\mp A\omega\frac{\partial^2 h}{\partial \phi^2}\right\}(\mp\omega) = (\mp)(\mp)A\omega^2\frac{\partial^2 h}{\partial \phi^2}$$

$$= A\omega^2\frac{\partial^2 h}{\partial \phi^2}$$

3RD PLUG THESE RESULTS INTO THE WAVE EQUATION

$$\left\{\frac{\partial^2 y}{\partial x^2} = Ak^2\frac{\partial^2 h}{\partial \phi^2}\right\} = \frac{1}{v^2}\left\{\frac{\partial^2 y}{\partial t^2} = A\omega^2\frac{\partial^2 h}{\partial \phi^2}\right\}$$

$\Rightarrow k^2 = \frac{\omega^2}{v^2}$ (AFTER CANCELING $A\,\partial^2 h/\partial\phi^2$) $\Rightarrow v^2 = \frac{\omega^2}{k^2} = \frac{(2\pi f)^2}{(2\pi/\lambda)^2}$

or $v^2 = f^2\lambda^2$ or $v = f\lambda = \left(\frac{6}{30} = \frac{1}{5}\text{ m/s}\right)$ ✓ WAVE EQUATION IS SATISFIED!