STAEDTLER® No. 937 811E Engineer's Computation Pad

••7 In 1654 Otto von Guericke, inventor of the air pump, gave a demonstration before the noblemen of the Holy Roman Empire in which two teams of eight horses could not pull apart two evacuated brass hemispheres. (a) Assuming the hemispheres have (strong) thin walls, so that R in Fig. 14-31 may be considered both the inside and outside radius, show that the force $\vec{F}$ required to pull apart the hemispheres has magnitude $F = \pi R^2 \Delta p$, where Δp is the difference between the pressures outside and inside the sphere. (b) Taking R as 30 cm, the inside pressure as 0.10 atm, and the outside pressure as 1.00 atm, find the force magnitude the teams of horses would have had to exert to pull apart the hemispheres. (c) Explain why one team of horses could have proved the point just as well if the hemispheres were attached to a sturdy wall.

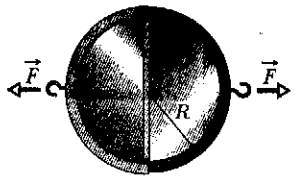
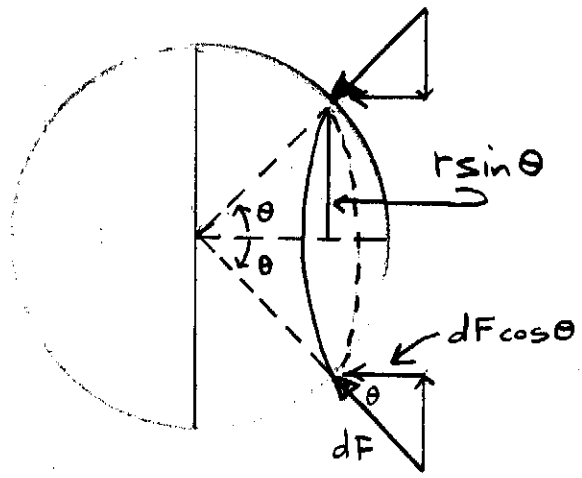


FIG. 14-31 Problem 7.





THE VERTICAL COMPONENT IN THE ABOVE DIAGRAM CANCELS BY SYMMETRY. THE NET PRESSURE FORCE ON A RING OF HEMISPHERE:

$$dF@\theta = \Delta P\, dA \cos\theta$$

WHERE dA = THE AREA OF THE RING = LENGTH × WIDTH $= (2\pi R \sin\theta) R\, d\theta$

$$F^{on}_{HEMISPHERE} = \int_{\theta=0}^{\theta=90^\circ} dF = \Delta P \cos\theta\, dA$$

$$= \Delta P\, 2\pi R^2 \int_0^{90^\circ} \sin\theta \cos\theta\, d\theta$$

$$\frac{d\cos\theta}{d\theta} = -\sin\theta \Rightarrow d\cos\theta = -\sin\theta\, d\theta$$

LET $y = \cos\theta \Rightarrow dy = -\sin\theta\, d\theta \Rightarrow F^{on}_{HEM} = \Delta P\, 2\pi R^2 \int_{y=1=\cos 0}^{y=0=\cos 90^\circ} -y\, dy$

a) $$F^{on}_{HEM} = \Delta P\, 2\pi R^2 \int_0^1 y\, dy = \Delta P\, 2\pi R^2 \frac{y^2}{2}\Big|_0^1 = \Delta P\, 2\pi R^2 \frac{1}{2} = \pi R^2 \Delta P$$

b) $$F^{on}_{HEM} = \pi (0.30^2)\left[1.01\times10^5 \tfrac{N}{m^2} - 0.1\left(1.01\times10^5 \tfrac{N}{m^2}\right)\right] = 0.90\,\pi R^2 \left[ATM = 1.05\times10^5 \tfrac{N}{m^2}\right]$$
$$= 2.6\times10^4\ N$$

c) BY NEWTON'S 3RD LAW OF MOTION (& EQUILIBRIUM ANALYSIS) THE FORCE OF THE WALL ON THE LEFT HEMISPHERE IS THE SAME AS THE FORCE ON THE RIGHT HEMISPHERE BY ONE TEAM OF HORSES!

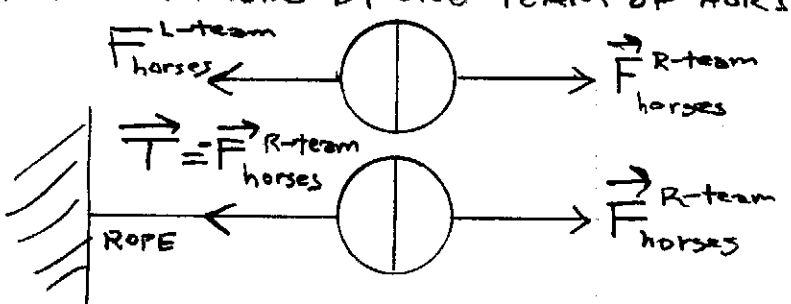


FIGURE 14.2 The famous "Magdeburg hemispheres" experiment of 1654, demonstrating atmosphere pressure. Two teams of horses couldn't pull the evacuated hemispheres apart. Were the hemispheres sucked together or pushed together? By what?