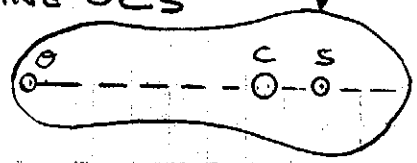
STAEDTLER® No. 937 811E Engineer's Computation Pad

111 The center of oscillation of a physical pendulum has this interesting property: If an impulse (assumed horizontal and in the plane of oscillation) acts at the center of oscillation, no oscillations are felt at the point of support. Baseball players (and players of many other sports) know that unless the ball hits the bat at this point (called the "sweet spot" by athletes), the oscillations due to the impact will sting their hands. To prove this property, let the stick in Fig. 15-11*a* simulate a baseball bat. Suppose that a horizontal force $\vec{F}$ (due to impact with the ball) acts toward the right at *P*, the center of oscillation. The batter is assumed to hold the bat at *O*, the pivot point of the stick. (a) What acceleration does the point *O* undergo as a result of $\vec{F}$? (b) What angular acceleration is produced by $\vec{F}$ about the center of mass of the stick? (c) As a result of the angular acceleration in (b), what linear acceleration does point *O* undergo? (d) Considering the magnitudes and directions of the accelerations in (a) and (c), convince yourself that *P* is indeed the "sweet spot."

I WILL TREAT A GENERAL PHYSICAL PENDULUM (BODY) AS A SPECIAL CASE (THEN THE STICK WILL FOLLOW)

IMPULSIVE FORCE $\vec{F}_{Av}\Delta t = \Delta\vec{p}$ APPLIED ⊥ TO LINE OCS



INITIALLY AT REST
OC = h
OS = L_o

THE IMPULSIVE FORCE CAUSES THE COM TO ACCELERATE IN THE DIRECTION OF THE FORCE!

THE RESULT IS THE OBJECT'S COM MOVES ALONG A STRAIGHT LINE WITH SOME MOMENTUM $\vec{P}_{total} = m\vec{v}_{com}$ GAINED AS A RESULT OF THE IMPULSE $\vec{F}_{Av}\Delta t$. THE OBJECT WILL ALSO ROTATE ABOUT ITS COM SINCE THE IMPULSE RESULTED IN AN IMPULSIVE TORQUE ABOUT THE COM (MOMENT ARM FROM COM ≠ 0).

$$\vec{F}_{net} = \frac{d\vec{p}}{dt} \Rightarrow \vec{F}_{Av}\Delta t = \Delta\vec{P}_{total} = \vec{P}^{tot}_{fin} - \vec{P}^{tot}_{in} = m\vec{v}_{com} = m\vec{v}_c \text{ in direction of } \vec{F}$$

$$\vec{\tau}^{net}_{com} = \frac{d\vec{L}}{dt} \Rightarrow \vec{\tau}^{net}_{com\,Av}\Delta t = \Delta\vec{L} = \vec{L}^{tot}_{fin} - \vec{L}^{tot}_{initial}$$

$$\vec{v}_{com} \equiv \vec{v}_c = \frac{\vec{F}_{Av}\Delta t}{m}$$

$$\Rightarrow \vec{r} \times \vec{F}_{Av}\Delta t = I_c\vec{\omega}_f - I_c(\vec{\omega}_{in} \equiv 0) \Rightarrow (r = CS = L_o - S)$$

$$\Rightarrow (r = CS = L_o - h)F_{Av}\Delta t \sin 90^\circ \text{ into page} = I_c\vec{\omega}_c \quad (\vec{\omega}_f = \vec{\omega}^{\text{about}}_{com} = \vec{\omega}_c)$$

$$\Rightarrow \vec{\omega}_c = [(L_o - h)F_{Av}\Delta t / I_c] \text{ into page}$$

IN THE COM FRAME OF REFERENCE THIS ROTATION ABOUT THE COM CAUSES POINTS IN THE BODY TO HAVE TANGENTIAL VELOCITIES IN PROPORTION TO THEIR DISTANCE FROM THE COM. i.e. $v_{\text{in com frame}} = r\omega_c$

IN OUR INERTIAL (LABORATORY) FRAME OF REFERENCE THESE ROTATIONAL VELOCITIES ABOUT THE COM ADD TO $\vec{v}_{com}$ IF THEY ARE TO THE RIGHT OF THE COM IN THE DIAGRAM ABOVE & THEY SUBTRACT IF THEY ARE TO THE LEFT OF THE COM.

i.e. $$v^{\text{of a point}}_{\text{relative to us}} = v_c \pm r\omega_c = \frac{F_{Av}\Delta t}{m} \pm \frac{r(L_o - h)F_{Av}\Delta t}{I_c}$$

$$(L_o - h)/I_c = (I_o/mh - h)/I_c \overset{\text{PARALLEL AXIS THEOREM}}{=} [(I_c + mh^2)/mh - h]/I_c = 1/mh$$

$$\Rightarrow v^{\text{pt @ r from com}}_{\text{relative to us}} = \left[1 \pm mr/mh\right]\frac{F_{Av}\Delta t}{m} = \left[1 \pm r/h\right]v_c$$

$\Rightarrow$ FOR POINT O (WHERE $r = h$) $v_o = [1 - 1]v_c \equiv 0$

$\Rightarrow$ NO MOMENTUM WILL BE TRANSFERED TO THE BATTER'S HANDS (@ O) $\Rightarrow$ NO STING (SINCE NO FORCE) $\Rightarrow$ SWEET SPOT!