STAEDTLER® No. 937 811E Engineer's Computation Pad

••16 At a certain harbor, the tides cause the ocean surface to rise and fall a distance d (from highest level to lowest level) in simple harmonic motion, with a period of 12.5 h. How long does it take for the water to fall a distance $0.250d$ from its highest level?

THE HEIGHT OF THE WATER RELATIVE TO MIDWAY BETWEEN MEAN HIGH & LOW WATER HEIGHT (THIS IS AVERAGED OVER THE 19 YEAR OR 235 LUNATION (SYNODIC MONTH) METON CYCLE) IS: (SHM IS GENERALLY AN APPROXIMATION)

$y(t) = y_m \cos(\omega t + \phi)$ MAY AS WELL SET $\phi = 0$ SO THAT HIGH TIDE OCCURS FIRST @ $t = 0$

HIGH WATER LEVEL @ HIGH TIDE IS $y = y_m = y_{max}$ above $y = 0$

LOW WATER LEVEL @ LOW TIDE IS $y = -y_m$ (below $y = 0$)

(AT WRIGHTSVILLE BEACH $y_m = 1.9$ feet $= 0.58$ m $= 1.16/2$ m)

(AT W-BEACH LOW TIDE OCCURS 1.5 HRS AFTER THE MOON CROSSES THE MERIDIAN (ACCURACY $\sim \pm 1/2$ hr.))

$d = 2y_m$ & WE WANT TO KNOW WHEN THE WATER LEVEL HAS DROPPED A DISTANCE fd (THIS PROBLEM HAS $f = 1/4$) FROM HIGH $\Rightarrow y(t_f) = y_m - f 2y_m$

THE TIDES REPEAT AFTER A PERIOD $= T$ ie.

$y(t=0) = y_m$ & $y(t=T) = y_m \cos \omega T = y_m$

$\Rightarrow \omega T = 2\pi \Rightarrow T = 2\pi/\omega = 12.5 \text{ hrs} \Rightarrow \omega = \frac{2\pi}{T} = 0.503 \frac{\text{RAD}}{\text{hr}}$

$y(t_f) = (1 - 2f) y_m = y_m \cos \omega t_f$

$\Rightarrow t_f =$ TIME FOR WATER TO FALL fd FROM HIGH $= \dfrac{\cos^{-1}(1-2f)}{\omega}$

or $t_f = T \dfrac{\cos^{-1}(1-2f)}{2\pi} \overset{f=1/4}{=\!=} 0.167\,T = T/6 = 2.08 \text{ hr}$

$= T \dfrac{\cos^{-1}(1-2f)}{360}$ if YOUR CALCULATOR IS SET FOR DEGREES

