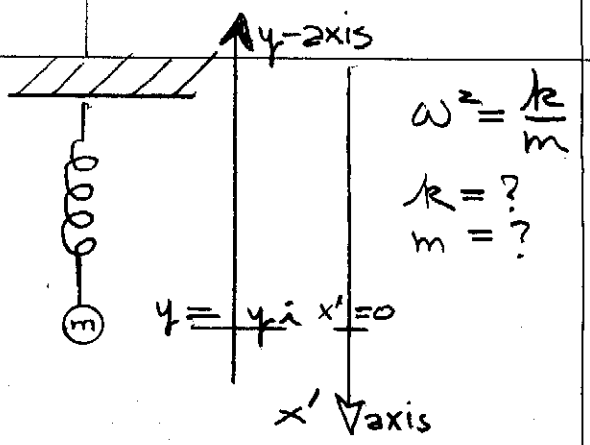
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•••37 A massless spring hangs from the ceiling with a small object attached to its lower end. The object is initially held at rest in a position y_i such that the spring is at its rest length. The object is then released from y_i and oscillates up and down, with its lowest position being 10 cm below y_i. (a) What is the frequency of the oscillation? (b) What is the speed of the object when it is 8.0 cm below the initial position? (c) An object of mass 300 g is attached to the first object, after which the system oscillates with half the original frequency. What is the mass of the first object? (d) How far below y_i is the new equilibrium (rest) position with both objects attached to the spring?



$y_i = y_{initial}$ = THE UNSTRETCHED SPRING POSITION OF MASS m

$-(y - y_i) = x'$ = THE STRETCH OF THE SPRING FROM ITS UNSTRETCHED LENGTH.

CONSERVATION OF ENERGY: $E_{TOTAL} = K + U_G + U_S = \text{CONSTANT}$

$$E_i = E_f \Rightarrow \frac{1}{2}mv_i^2 + mgy_i + \frac{1}{2}kx_i'^2 = \frac{1}{2}mv_f^2 + mgy_f + \frac{1}{2}kx_f'^2$$

$$\Rightarrow 0 + mgy_i + \frac{1}{2}k(0)^2 = 0 + mgy_f + \frac{1}{2}k(y_i - y_f)^2$$

$$\Rightarrow mg(y_i - y_f) = \frac{1}{2}k(y_i - y_f)^2 \Rightarrow \frac{k}{m} = 2g/(y_i - y_f)$$

$$\omega^2_{original} = \omega_0^2 = (2\pi f_0)^2 = \left(2\pi/T_0\right)^2 = k/m = 2(9.8)/0.1m = 196 \frac{RAD^2}{S^2}$$

a) $\Rightarrow \omega_0 = 14 \frac{RAD}{S} \Rightarrow f_0 = \frac{\omega_0}{2\pi} = 2.23 \frac{osc}{S} \Rightarrow T_0 = \frac{1}{f_0} = 0.449\,S$

b) $E_i = E_f \Rightarrow 0 + mgy_i + 0 = \frac{1}{2}mv_f^2 + mgy_f + \frac{1}{2}k(y_i - y_f)^2$

$$\Rightarrow v_f^2 = 2g(y_i - y_f) - \frac{k}{m}(y_i - y_f)^2 = 2(9.8)(0.08) - 196(0.08)^2 = 0.314 \frac{m^2}{S^2}$$

$$\Rightarrow v_f = 0.56\ m/s$$

c) THE MASS BECOMES $m_{new} = m + m'$ WHERE $m' = 300g$

THE FREQUENCY BECOMES $\omega_n = \frac{1}{2}\omega_0 \Rightarrow \omega_n^2 = \frac{1}{4}\omega_0^2$

$$\Rightarrow \frac{k}{m_n} = \frac{1}{4}\frac{k}{m} \Rightarrow m_n = m + m' = 4m \Rightarrow m' = 3m = 300g$$

$$\Rightarrow m = 100g = 0.1\,kg$$

d) EQUILIBRIUM $\Rightarrow \vec{F}^{net}_{on\,m_n} = 0 \Rightarrow mg = k\left|x' = (y_i - y_E)\right|$

$$\omega_0^2 = \frac{k}{m} = 196 = \frac{k}{.1kg} \Rightarrow k = 19.6$$

$$y_i - y_E = mg/k = g/(k/m) = \frac{9.8}{196} = 0.05m = 5\,cm$$

ORIGINAL

$$y_i - y_E = m_n g/k = (0.3 + 0.1)\frac{9.8}{19.6} = \frac{0.4(9.8)}{19.6} = 0.2m = 20\,cm$$

WITH BOTH MASSES

FIND THE POSITION AS A FUNCTION OF THE TIME FOR A MASS m SUSPENDED FROM A VERTICAL SPRING OF SPRING CONSTANT k

THE UNSTRETCHED EQUILIBRIUM POSITION OF SPRING @ $y = y_i$ or $x' \equiv 0$

EQUILIBRIUM POSITION OF SUSPENDED MASS MASS, m, VIA $F^{net}_{on\ m} \equiv 0$

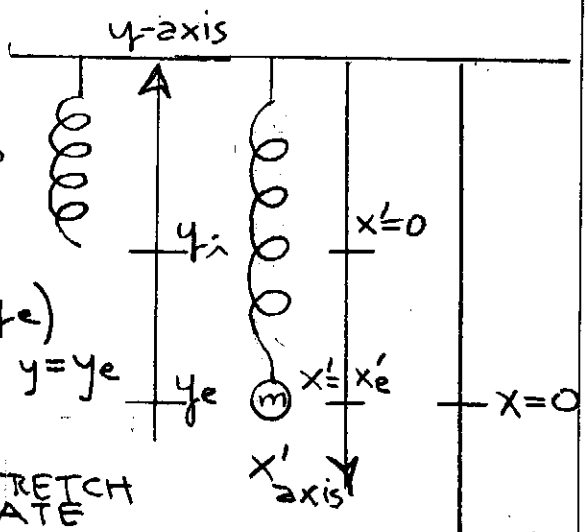
$\Rightarrow mg\hat{x}' + (-kx'_e\hat{x}') = 0 \rightarrow x'_e = \frac{mg}{k}$

$\Rightarrow mg = kx'_e$ or $mg = k(y_i - y_e)$

Let $x=0$ correspond to $x' = x'_e$ or $y = y_e$

$\Rightarrow x = x' - x'_e = x' - mg/k$

$\Rightarrow x' = x + mg/k$ = SPRING STRETCH COORDINATE



NEWTON'S 2ND LAW – THE Eqn of Motion

$\vec{F}^{net}_{on\ m} = m\vec{a}_{of\ m} \Rightarrow mg\hat{x} + \left[-kx' = -k\left(x + \frac{mg}{k}\right)\right]\hat{x} = m\frac{d^2x}{dt^2}\hat{x}$

$\Rightarrow mg - mg - kx = m\frac{d^2x}{dt^2} \Rightarrow \frac{d^2x}{dt^2} = -\omega^2 x \quad \omega^2 \equiv k/m$

$\Rightarrow x(t) = x_m \cos(\omega t + \phi)$ (just like GRAVITY WAS NOT INVOLVED!)

CONSERVATION OF ENERGY: E_{total} = CONSTANT

$E_{total} = K + U_g + U_s = \frac{1}{2}mv^2 + mgy + \frac{1}{2}kx'^2$

WE HAVE THE CONVERSIONS: $y - y_i = -x'$ & $x' = x + \frac{mg}{k}$

$\Rightarrow y = y_i - x' = y_i - x - \frac{mg}{k}$

$\Rightarrow E_{total} = \frac{1}{2}mv^2 + mg\left[y_i - x - \frac{mg}{k}\right] + \frac{1}{2}k\left(x + \frac{mg}{k}\right)^2$

$= \frac{1}{2}mv^2 + mgy_i - mgx - \frac{(mg)^2}{k} + \frac{1}{2}k\left[x^2 + \frac{2mg}{k}x + \frac{(mg)^2}{k^2}\right]$

$= \frac{1}{2}mv^2 + \frac{1}{2}kx^2 \underbrace{-mgx + mgx}_{0} + \underbrace{mgy_i - \frac{(mg)^2}{2k}}_{\text{CONSTANT}}$

$\Rightarrow E_{TOTAL} = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$ GIVES CORRECT PHYSICS (AS IF GRAVITY PLAYS NO ROLE)

REASON: GRAVITATIONAL FORCE IS CONSTANT AND ALWAYS BALANCED OUT BY THE CONSTANT kx'_e SPRING FORCE WHICH EFFECTIVELY DEFINES A NEW EQUILIBRIUM POSITION AT $x \equiv 0$

(ie $F = kx = kx' - (mg = kx'_e)$)

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