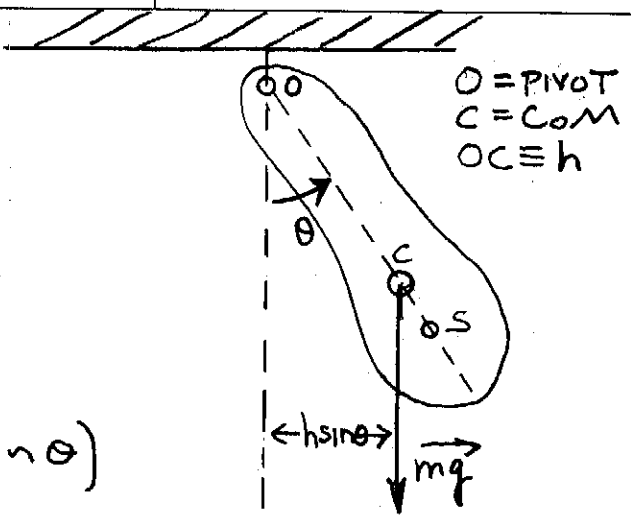
STAEDTLER® No. 937 811E Engineer's Computation Pad

•42 In Sample Problem 15-5, we saw that a physical pendulum has a center of oscillation at distance $2L/3$ from its point of suspension. Show that the distance between the point of suspension and the center of oscillation for a physical pendulum of any form is I/mh, where I and h have the meanings assigned to them in Eq. 15-29 and m is the mass of the pendulum.



$$\vec{\tau}_{\text{about O}}^{\text{net}} = I_O \vec{\alpha}_{\text{about O}}$$

$$\tau_O^{\text{net}} = \tau_O^{\text{FORCE @ PIVOT}} + \tau_O^{\text{GRAVITY}}$$

$$= F^{@\text{PIVOT}}(0) + -mg(h\sin\theta)$$

$$\Rightarrow -mgh\sin\theta = I_O \frac{d^2\theta}{dt^2}$$

FOR SMALL ANGLES $\sin\theta \approx \theta$ IN RADIAN MEASURE

$$\Rightarrow \frac{d^2\theta}{dt^2} = \frac{-mgh}{I_O}\theta \quad \text{DEFINE } \omega^2 \equiv \frac{mgh}{I_O}$$

$$\Rightarrow \frac{d^2\theta}{dt^2} = -\omega^2\theta \Rightarrow \theta(t) = \theta_m\cos(\omega t + \phi)$$

$$\theta(t=0) = \theta_m\cos\phi \;\&\; \theta(t=T) = \theta_m\cos(\omega T + \phi) = \theta_m\cos\phi$$

IF $\omega T = 2\pi \Rightarrow T = 2\pi/\omega$

$$T_{\text{Physical}} = 2\pi\sqrt{\frac{I_O}{mgh}}$$

A SIMPLE PENDULUM HAS $I_O = mL^2$ & $h = L$

$$\Rightarrow T_{\text{simple}} = 2\pi\sqrt{\frac{mL^2}{mgL}} = 2\pi\sqrt{\frac{L}{g}} \;\checkmark$$

DEFINITION OF CENTER OF OSCILLATION = SWEET SPOT
(also called CENTER OF PERCUSSION)
= S: THE POINT ALONG THE LINE CONNECTING THE PIVOT (O) & THE CENTER OF MASS (C) THAT IS A DISTANCE L_O FROM THE PIVOT, WHERE L_O IS THE LENGTH OF THE EQUIVALENT SIMPLE PENDULUM IN THE FOLLOWING "PERIOD SENCE"

$$T_{\text{Physical}} = 2\pi\sqrt{\frac{I_O}{mgh}} = 2\pi\sqrt{\frac{L_O}{g}} = T_{\substack{\text{EQUIVALENT SIMPLE PENDULUM} \\ \text{WITH THE SAME PERIOD}}}$$

$$\Rightarrow L_O = I_O/mh$$

IT DEPENDS ON THE LOCATION OF THE PIVOT POINT & THE COM

$L_O = OS$