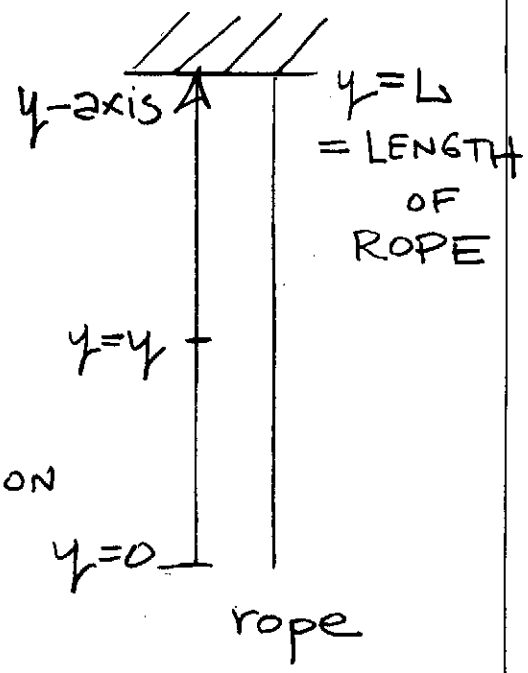
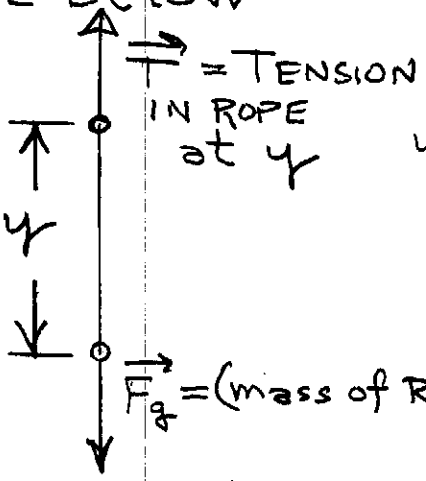
STAEDTLER® No. 937 811E Engineer's Computation Pad

•••25 A uniform rope of mass m and length L hangs from a ceiling. (a) Show that the speed of a transverse wave on the rope is a function of y, the distance from the lower end, and is given by $v = \sqrt{gy}$. (b) Show that the time a transverse wave takes to travel the length of the rope is given by $t = 2\sqrt{L/g}$.



APPLY THE CONDITION OF EQUILIBRIUM TO THE LENGTH OF ROPE BELOW $y=y$



$\vec{F}_{net\ on\ ROPE\ SECTION} \equiv 0$

$\Rightarrow \vec{T}_{@y} + m_{\substack{SECTION \\ LENGTH\ y}} \vec{g} \equiv 0$

THE MASS PER UNIT LENGTH OF THE ROPE $= \mu = m/L \Rightarrow$ MASS OF ROPE SEGMENT $= \mu y = \frac{my}{L}$

EQUILIBRIUM $\Rightarrow T = m^{SEGMENT}_{LENGTH\ y} g = \left(\frac{my}{L}\right) g = \mu y g$

SPEED OF WAVE ON ROPE $= v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{myg}{L} \Big/ m/L}$

a) $v = \sqrt{\mu y g / \mu} = \sqrt{gy}$

b) NOTE: $v = \frac{dy}{dt} \Rightarrow dt = dy/v$

$\Rightarrow \int_{t=0}^{t=T=\text{time to travel } L} dt = \int_{y=0}^{y=L} \frac{1}{v}\, dy = \int_0^L \frac{1}{\sqrt{gy}}\, dy$

$= \frac{1}{\sqrt{g}} \int_0^L y^{-1/2}\, dy = \frac{1}{\sqrt{g}} \left. \frac{y^{-1/2+1}}{-\frac{1}{2}+1} \right|_0^L$

$= \frac{2}{\sqrt{g}} \sqrt{y} \Big|_0^L = 2\sqrt{L/g}$