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sec. 16-7 Energy and Power of a Wave Traveling Along a String

••27 A sinusoidal wave is sent along a string with a linear density of 2.0 g/m. As it travels, the kinetic energies of the mass elements along the string vary. Figure 16-39*a* gives

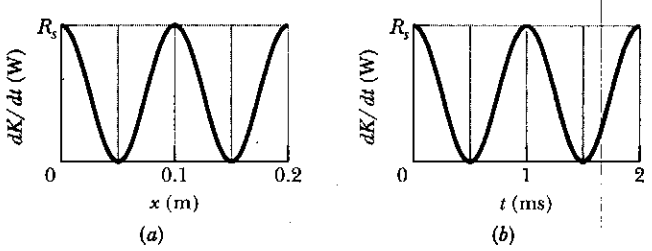


FIG. 16-39 Problem 27.

the rate dK/dt at which kinetic energy passes through the string elements at a particular instant, plotted as a function of distance x along the string. Figure 16-39*b* is similar except that it gives the rate at which kinetic energy passes through a particular mass element (at a particular location), plotted as a function of time t. For both figures, the scale on the vertical (rate) axis is set by $R_s = 10$ W. What is the amplitude of the wave?

THE KINETIC ENERGY OF AN INFINITESIMAL MASS ELEMENT $dm = \mu dx$ OF THE STRING IS INFINITESIMAL, IE $= dK$ AND IS OBTAINED VIA ITS VELOCITY $v_y = u = \frac{\partial y_m}{\partial t} = \frac{\partial}{\partial t}\left(y_m \sin(kx \pm \omega t + \phi)\right)$

$= v_y = \pm \omega y_m \cos(kx - \omega t + \phi)$

$dK = \frac{1}{2}(dm = \mu dx)\,\omega^2 y_m^2 \cos^2(kx - \omega t + \phi)$

= KINETIC ENERGY PART OF THE TOTAL ENERGY = dE OF THE SHO ELEMENT $dm = \mu dx$

THE TIME AVERAGE IS:

$\langle dK \rangle = \frac{1}{2}\mu dx\,\omega^2 y_m^2 \left(\frac{1}{2}\right)$

WHICH (WE REMEMBER) IS EQUALLY SHARED BY A POTENTIAL ENERGY COMPONENT TO THE TOTAL ENERGY $\langle dU \rangle = \frac{1}{4}\mu dx\,\omega^2 y_m^2$

$\Rightarrow \langle dE \rangle = dE = \frac{1}{2}\mu dx\,\omega^2 y_m^2$

AS THE WAVE FORM (DISTURBANCE IN THE MEDIUM) TRAVELS AT SPEED $v = v_x = \lambda f = \omega/k$ THIS SHM-ENERGY IS PROGRESSIVELY INDUCED IN THE MEDIUM'S ELEMENTS $dm = \mu dx$ AS THE WAVE PROGRESSES (IE. SHM IS SEQUENTIALLY TURNED ON) - THIS CORRESPONDS TO A TRANSFER OF ENERGY PER UNIT TIME - THAT ULTIMATELY CAME FROM THE SOURCE OF THE WAVE DISTURBANCE.

$\text{POWER} = P_{AVG} = \frac{\langle dE \rangle}{dt} = \left\langle \frac{dK}{dt} \right\rangle + \left\langle \frac{dU}{dt} \right\rangle = \frac{dE_{total}}{dt} = \frac{\frac{1}{2}\mu dx\,\omega^2 y_m^2}{dt}$

$P_{AVG} = \frac{1}{2}\mu v \omega^2 y_m^2$ where $v = v_x = \frac{dx}{dt} = \lambda f = \omega/k$

THE GRAPHS ARE OF $\frac{dK}{dt} = \frac{1}{2}\mu v \omega^2 y_m^2 \cos^2(kx \pm \omega t + \phi)$

or $\frac{dK}{dt} = \frac{1}{2}\mu v \omega^2 y_m^2 \cos^2\left[2\pi\left(\frac{x}{\lambda}\right) \pm 2\pi\left(\frac{t}{T}\right)\right]$ λ = PERIOD IN SPACE, T = PERIOD IN TIME

THE 1ST GRAPH GIVES THE PERIOD IN SPACE = WAVELENGTH = $0.2\text{m} = \lambda$

THE 2ND GRAPH GIVES THE PERIOD IN TIME = PERIOD = $0.002\text{s} = T$

THE MAXIMUM VALUE OF $\frac{dK}{dt} = \frac{1}{2}\mu v \omega^2 y_m^2 = R_s = 10\text{W}$

$\Rightarrow \frac{1}{2}\mu\left(v = \frac{\lambda}{T}\right)\frac{4\pi^2}{T^2} y_m^2 = R_s \Rightarrow y_m^2 = T^3 R_s / (2\pi^2 \mu \lambda)$

$y_m = \sqrt{(.002)^3 10 / \left(2\pi^2 (0.002\,\text{kg/m})\,0.2\right)} = 0.0032\,\text{m} = 3.2\,\text{mm}$