99 A certain juggler usually tosses balls vertically to a height H. To what height must they be tossed if they are to spend twice as much time in the air? [redacted]

THE TIMINGS & VELOCITIES ARE SYMMETRIC

WITH RESPECT TO THE MIDPOINT IN TIME — THE TOP OF THE TRAJECTORY WHERE $v = 0$.

THE TIME TO FALL FROM REST AT HEIGHT H IS

$$H = \frac{1}{2} a t^2 = \frac{1}{2} g t^2 \Rightarrow t_{\text{to fall } H} = \sqrt{\frac{2H}{g}}$$

TIME IN THE AIR FOR A HEIGHT $H = t_H$

$$= t_H = 2 t_{\text{to fall } H} = 2\sqrt{\frac{2H}{g}} = \sqrt{\frac{8H}{g}}$$

CONSIDER THROWING TO A HEIGHT H' SUCH THAT $t_{H'} = R\, t_H$

THEN $t_{H'} = \sqrt{\frac{8H'}{g}} = R\, t_H = R\sqrt{\frac{8H}{g}}$

$$\Rightarrow \sqrt{\frac{8H'}{g}} = R\sqrt{\frac{8H}{g}} \Rightarrow \sqrt{H'} = R\sqrt{H}$$

$$\Rightarrow H' = R^2 H \Rightarrow$$

ie R times more or less time

$\Rightarrow R^2$ times more or less distance

This case: $R = 2$ i.e. twice the time in the air

$\Rightarrow R^2 = 2^2 = 4$ times the height

EXTRA $\Rightarrow$ How much faster must it be thrown

$$v^2 = v_0^2 + 2a(y - y_0) \xrightarrow{v = 0 \text{ at top}} 0 = v_0^2 - 2gH$$

$$v_0 = \sqrt{2gH} \quad \& \quad v_0' = \sqrt{2gH'} = \sqrt{2gR^2H}$$

$$v_0' = R\sqrt{2gH} = R\, v_0$$

$\Rightarrow R$ times more or less time

$\Rightarrow R^2$ times more or less distance to throw up

$\Rightarrow R$ times more or less speed

(more for $R > 0$ & less for $R < 0$)

STAEDTLER® No. 937 811E Engineer's Computation Pad