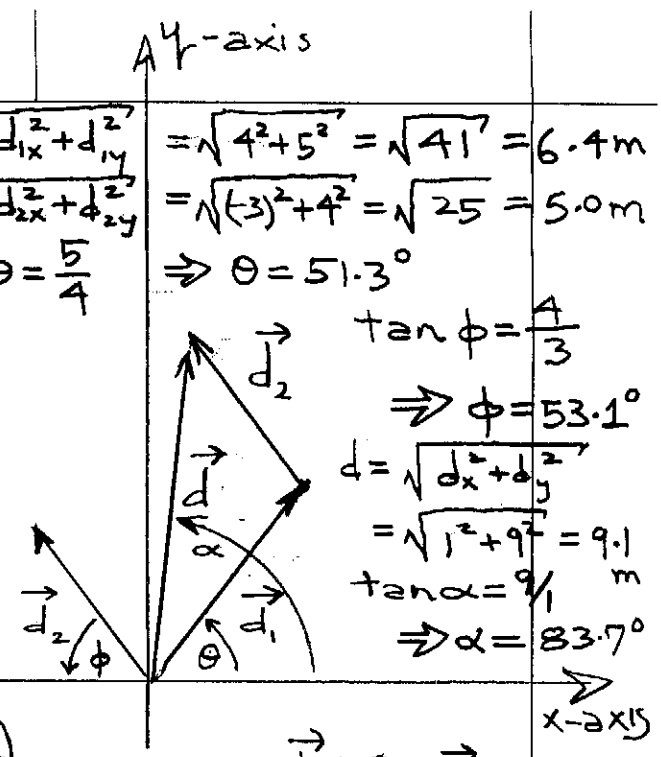
STAEDTLER® No. 937 811E Engineer's Computation Pad

••44 In a meeting of mimes, mime 1 goes through a displacement $\vec{d}_1 = (4.0\text{ m})\hat{i} + (5.0\text{ m})\hat{j}$ and mime 2 goes through a displacement $\vec{d}_2 = (-3.0\text{ m})\hat{i} + (4.0\text{ m})\hat{j}$. What are (a) $\vec{d}_1 \times \vec{d}_2$, (b) $\vec{d}_1 \cdot \vec{d}_2$, (c) $(\vec{d}_1 + \vec{d}_2) \cdot \vec{d}_2$, and (d) the component of $\vec{d}_1$ along the direction of $\vec{d}_2$? (*Hint:* For (d), see Eq. 3-20 and Fig. 3-20.)



$d_1 = \sqrt{d_{1x}^2 + d_{1y}^2} = \sqrt{4^2+5^2} = \sqrt{41} = 6.4\text{ m}$

$d_2 = \sqrt{d_{2x}^2 + d_{2y}^2} = \sqrt{(-3)^2+4^2} = \sqrt{25} = 5.0\text{ m}$

$\tan\theta = \frac{5}{4} \Rightarrow \theta = 51.3°$

$\tan\phi = \frac{4}{3} \Rightarrow \phi = 53.1°$

$\vec{d}_1 = 4\hat{x} + 5\hat{y} = d_{1x}\hat{x} + d_{1y}\hat{y}$

$\vec{d}_2 = -3\hat{x} + 4\hat{y} = d_{2x}\hat{x} + d_{2y}\hat{y}$

$\vec{d} = \vec{d}_1 + \vec{d}_2 = d_x\hat{x} + d_y\hat{y}$

$= (d_{1x}+d_{2x})\hat{x} + (d_{1y}+d_{2y})\hat{y}$

$\vec{d} = 1\hat{x} + 9\hat{y}$

$d = \sqrt{d_x^2 + d_y^2} = \sqrt{1^2+9^2} = 9.1\text{ m}$

$\tan\alpha = \frac{9}{1} \Rightarrow \alpha = 83.7°$

$\vec{d}_1 \times \vec{d}_2 = d_1 d_2 \sin\measuredangle(\vec{d}_1, \vec{d}_2)(\hat{z}=\hat{k})$

$\measuredangle(\vec{d}_1, \vec{d}_2) = 180 - \theta - \phi = 75.6°$

$|\vec{d}_1 \times \vec{d}_2| = (6.4)(5.0)\sin 75.6° = 31.0$

= Area of Parallelogram

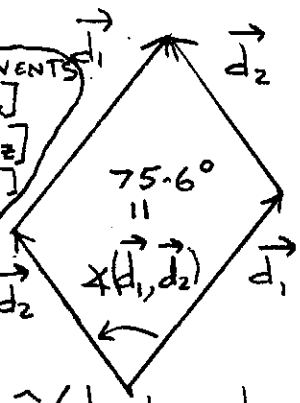
= BASE × HEIGHT

$= d_1(d_2 \sin 75.6)$ or $d_2(d_1 \sin 75.6)$

DO IT WITH COMPONENTS

$= \hat{x}[d_{1y}d_{2z} - d_{1z}d_{2y}] + \hat{y}[d_{1z}d_{2x} - d_{1x}d_{2z}] + \hat{z}[d_{1x}d_{2y} - d_{1y}d_{2x}]$

$= 0\hat{x} + 0\hat{y} + [4(4) - (5)(-3)]\hat{z} = [16 - -15 = 31]\hat{z}$



$\vec{d}_1 \times \vec{d}_2 = \hat{x}(d_{1y}d_{2z} - d_{1z}d_{2y}) + \hat{y}(d_{1z}d_{2x} - d_{1x}d_{2z}) + \hat{z}(d_{1x}d_{2y} - d_{1y}d_{2x})$

$= 0\hat{x} + 0\hat{y} + [(4)(4) - (5)(-3)]\hat{z} = [(16 - -15) = 31]\hat{z}$ ✓

$\vec{d}_1 \cdot \vec{d}_2 = d_1 d_2 \cos\measuredangle(\vec{d}_1, \vec{d}_2) = (6.4)(5.0)\cos 75.6° = 8.0\text{ m}^2$

DO IT WITH COMPONENTS $= d_{1x}d_{2x} + d_{1y}d_{2y} = 4(-3) + 5(4) = -12 + 20 = 8$ ✓

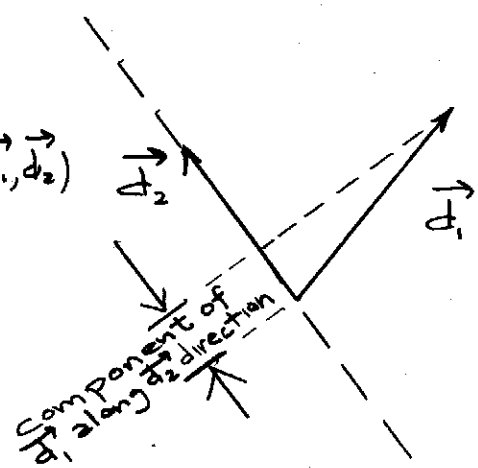
$(\vec{d}_1 + \vec{d}_2) \cdot \vec{d}_2 = \vec{d}_1 \cdot \vec{d}_2 + \vec{d}_2 \cdot \vec{d}_2 = 8.0 + d_2 d_2 \cos 0$

$= 8.0 + d_2^2 = 8.0 + 5.0^2 = 8 + 25 = 33\text{ m}^2$

Component of $\vec{d}_1$ along the direction of $\vec{d}_2$

$= \vec{d}_1 \cdot \left(\frac{\vec{d}_2}{d_2}\right) = \frac{\vec{d}_1 \cdot \vec{d}_2}{d_2} = \frac{d_1 d_2 \cos\measuredangle(\vec{d}_1, \vec{d}_2)}{d_2}$

$= d_1 \cos\measuredangle(\vec{d}_1, \vec{d}_2) = \frac{8.0}{5.0} = 1.6\text{ m}$



Component of $\vec{d}_2$ along the direction of $\vec{d}_1$

$= \vec{d}_2 \cdot \left(\frac{\vec{d}_1}{d_1}\right)$

Component of a Vector $\vec{v}$ in the direction of Unit Vector $\hat{u}$ $= \vec{v} \cdot \hat{u}$