STAEDTLER® No. 937 811E Engineer's Computation Pad

64 A wheel with a radius of 45.0 cm rolls without slipping along a horizontal floor (Fig. 3-39). At time t_1, the dot P painted on the rim of the wheel is at the point of contact between the wheel and the floor. At a later time t_2, the wheel has rolled through one-half of a revolution. What are (a) the magnitude and (b) the angle (relative to the floor) of the displacement of P?

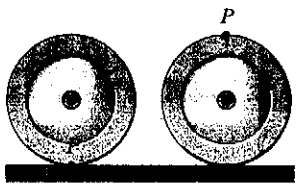
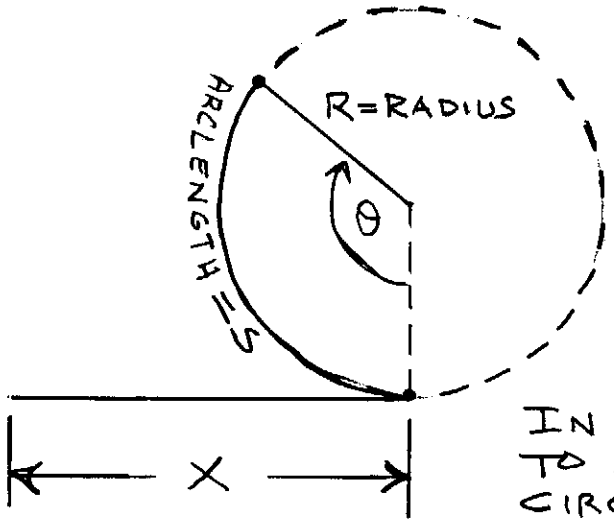


FIG. 3-39 Problem 64.

ROLLING WITHOUT SLIPPING

⇒ THE ANGLE TURNED THROUGH, MEASURED FROM THE CENTER OF THE WHEEL, IS RELATED TO THE DISTANCE X TRAVELED BY THE CENTER OF THE WHEEL (or the distance traveled on the surface)



DEFINITION OF ANGLE IN RADIANS

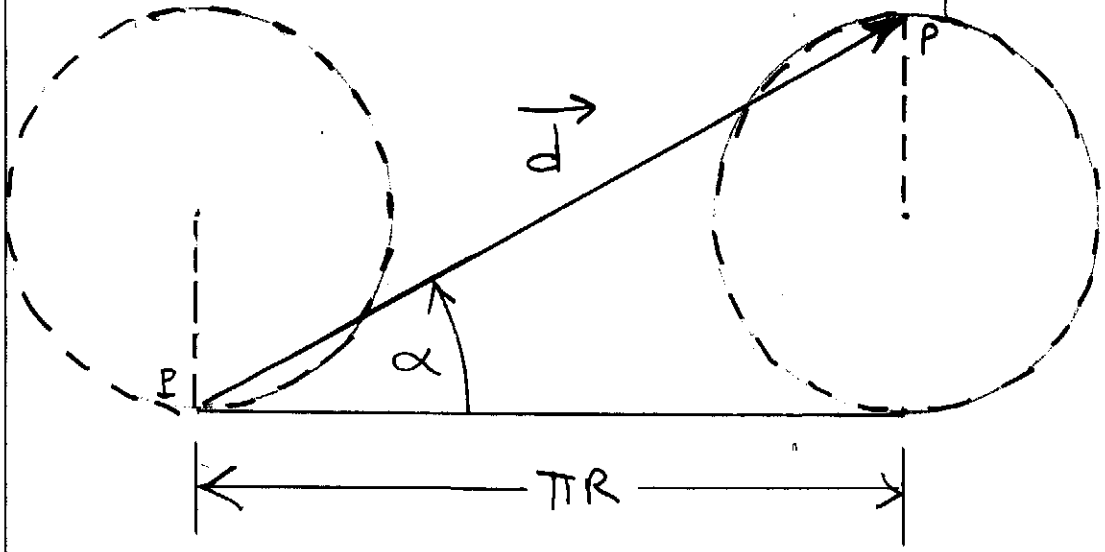
$$\theta \equiv \frac{S}{R} \Rightarrow S = R\theta$$

NO SLIP ⇒ X = DISTANCE TRAVELED = S

$$\Rightarrow X = R\theta$$

IN THIS EXAMPLE, IT IS EASY TO SEE THAT X = HALF OF CIRCUMFERENCE SINCE $\theta = 180°$

$$X = \frac{2\pi R}{2} = \pi R \quad \text{but check: } X = R\theta = R\left\{180\,\text{deg}\left(\frac{2\pi\,\text{Radians}}{360\,\text{deg}}\right)\right\} \checkmark$$



DIAMETER = 2R

(NOT QUITE TO SCALE)

$$\|\vec{d}\|^2 \xrightarrow[\text{THEOREM}]{\text{by Pythagorean}} (X=\pi R)^2 + (Y=2R)^2 = R^2(\pi^2+4)$$

$$\Rightarrow d = \|\vec{d}\| = 3.72R = 3.72(45\,\text{cm}) = 167\,\text{cm}$$

$$\tan\alpha = \frac{2R}{\pi R} = \frac{2}{\pi} \Rightarrow \alpha = 32.5°$$