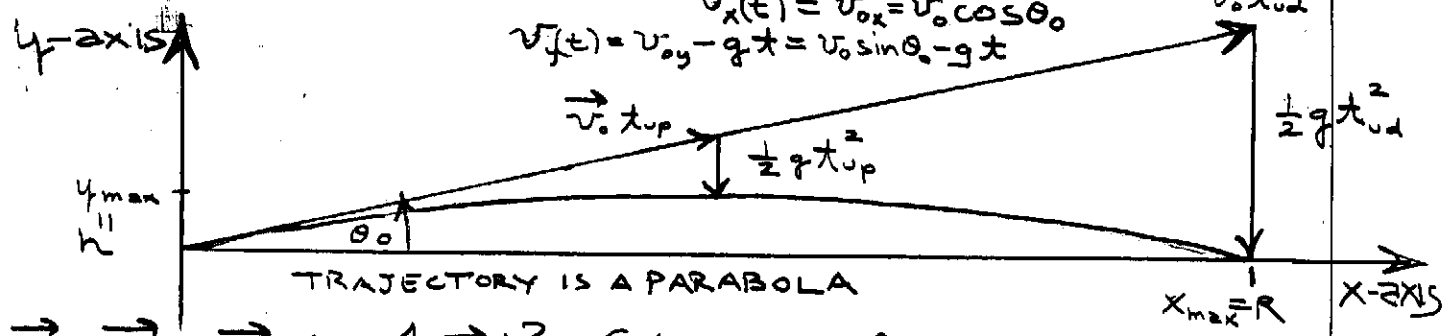
PROJECTILE MOTION

•23 The current world-record motorcycle jump is 77.0 m, set by Jason Renie. Assume that he left the take-off ramp at 12.0° to the horizontal and that the take-off and landing heights are the same. Neglecting air drag, determine his take-off speed.

$$\vec{v} = \vec{v}_0 + \vec{g}t = \left[(v_{0x} = v_0\cos\theta_0)\hat{x} + (v_{0y} = v_0\sin\theta_0)\hat{y}\right] + \left[0\hat{x} + -g\hat{y}\right]t$$

$$v_x(t) = v_{0x} = v_0\cos\theta_0$$

$$v_y(t) = v_{0y} - gt = v_0\sin\theta_0 - gt$$



$$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{g}t^2 = (x_0\hat{x} + y_0\hat{y}) + \left[(v_{0x} = v_0\cos\theta_0)\hat{x} + (v_{0y} = v_0\sin\theta_0)\hat{y}\right]t + \frac{1}{2}(0\hat{x} + -g\hat{y})t^2$$

$$\vec{r}(t) = x(t)\hat{x} + y(t)\hat{y}$$

EQUATE x & y Components

$$\Rightarrow x(t) = x_0 + v_{0x}t = x_0 + v_0\cos\theta_0 t = \text{UNIFORM MOTION}$$

$$y(t) = y_0 + v_{0y}t - \frac{1}{2}gt^2 = y_0 + v_0\sin\theta_0 t - \frac{1}{2}gt^2 \quad \text{UNIFORMLY ACCELERATED MOTION}$$

TIME TO GO UP = t_{up} via $v_{0y}(t_{up}) \equiv 0 \Rightarrow v_0\sin\theta_0 - gt_{up} = 0$

$$\Rightarrow t_{up} = v_{0y}/g = v_0\sin\theta_0/g$$

TIME TO GO UP & DOWN = t_{ud} via $y(t_{ud}) \equiv 0 \Rightarrow 0 = 0 + v_0\sin\theta_0 t_{ud} - \frac{1}{2}gt_{ud}^2$

$$\Rightarrow t_{ud} = 2v_{0y}/g = 2v_0\sin\theta_0/g = 2t_{up} \checkmark$$

$y_{max} = h$ via either $y(t_{up}) = y_{max}$ or $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$

or $v_y^2 = v_{0y}^2 - 2g(y - y_0)$ or $0 = (v_0\sin\theta_0)^2 - 2g(y_{max} - 0)$

$$\Rightarrow y_{max} = v_0^2\sin^2\theta_0/2g$$

RANGE = $x_{max} = x(t_{ud}) \equiv R$ via $x(t) = x_0 + v_{0x}t$

$$\Rightarrow x(t_{ud}) = R = 0 + v_0\cos\theta_0\,(2v_0\sin\theta_0/g)$$

$$= 2v_0^2\sin\theta_0\cos\theta_0/g$$

TRIG IDENTITY $2\sin\theta_0\cos\theta_0 = \sin 2\theta_0 \Rightarrow R = v_0^2\sin 2\theta_0/g$

IN THIS QUESTION WE ARE TO FIND v_0 GIVEN R & θ_0

$$\Rightarrow v_0 = \sqrt{Rg/\sin 2\theta_0} = \sqrt{77.0(9.80)/\sin 24^\circ}$$

$$= 43.1\ \text{m/s} = 43.1\ \text{m/s}\ \frac{60\ \text{mi/hr}}{26.8\ \text{m/s}} = 96.5\ \text{mi/hr}$$

STAEDTLER® No. 937 811E Engineer's Computation Pad