@ STAEDTLER® No. 937 811E Engineer's Computation Pad

••30 A trebuchet was a hurling machine built to attack the walls of a castle under siege. A large stone could be hurled against a wall to break apart the wall. The machine was not placed near the wall because then arrows could reach it from the castle wall. Instead, it was positioned so that the stone hit the wall during the second half of its flight. Suppose a stone is launched with a speed of $v_0 = 28.0$ m/s and at an angle of $\theta_0 = 40.0°$. What is the speed of the stone if it hits the wall (a) just as it reaches the top of its parabolic path and (b) when it has descended to half that height? (c) As a percentage, how much faster is it moving in part (b) than in part (a)?

WE ARE INTERESTED IN

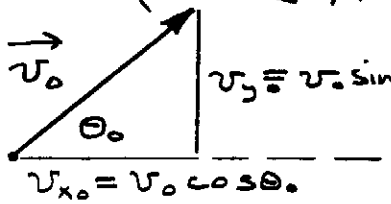
$\vec{v}(t) = \vec{v}_0 + \vec{a}t$ $\quad \vec{a} = \vec{g}$ = CONSTANT

$\vec{v} = \vec{v}_0 + \vec{g}t$

$(v_x\hat{x} + v_y\hat{y}) = (v_{0x}\hat{x} + v_{0y}\hat{y}) + (0\hat{x} + -g\hat{y})t$

$\Rightarrow v_x = v_{0x} = v_0\cos\theta_0$ = CONSTANT

& $v_y = v_{0y} - gt = v_0\sin\theta_0 - gt$ = UNIFORMLY CHANGING



(a) THE TOP OF THE TRAJECTORY

THE VELOCITY VECTOR IS HORIZONTAL SINCE IT IS TANGENT TO THE PARABOLIC TRAJECTORY

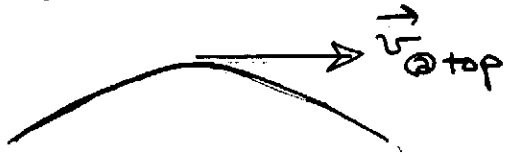
THEREFORE v_y(@ t_{up}, top of trajectory) $\equiv 0$

SO $\vec{v}_{top} = v_{0x}\hat{x}$

a) $v_{top} = v_{0x} = v_0\cos\theta_0$

$= 28.0\cos 40°$

$= 21.4\ \frac{m}{s}$



b) THE MOTION IS A LINEAR SUPERPOSITION OF CONSTANT VELOCITY MOTION IN THE x=HORIZONTAL & UNIFORMLY ACCELERATED MOTION IN THE y=VERTICAL

WE ANALYSE THE VERTICAL MOTION BY ITSELF, AND USE

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0) \Rightarrow v_y^2 = (v_0\sin\theta_0)^2 + 2(-g)(y - y_0)$

BECAUSE THIS TELLS US v_y WHEN WE KNOW y:

@ top, $y = h \Rightarrow 0^2 = (v_0\sin\theta_0)^2 - 2gh \Rightarrow h = (v_0\sin\theta_0)^2/2g$

@ half way $y = \frac{h}{2} = (v_0\sin\theta_0)^2/4g \Rightarrow$ USE THIS WITH

v_y^2 @ half way $= v_{0y}^2 - 2g\frac{h}{2} = (v_0\sin\theta_0)^2 - (v_0\sin\theta_0)^2/2$

$\Rightarrow v_y = \frac{v_0\sin\theta_0}{\sqrt{2}}$

IF WE CONSIDERED A FRACTION OF THE HEIGHT: $y = fh$ THEN

$v_y^2 = (v_0\sin\theta_0)^2 - 2gf\frac{(v_0\sin\theta_0)^2}{2g}$

$\Rightarrow v_y = v_0\sin\theta_0\sqrt{1-f}$ here $f = \frac{1}{2} \Rightarrow \sqrt{1-f} = \frac{1}{\sqrt{2}}$

$\vec{v} = v_x\hat{x} + v_y\hat{y} = v_0\cos\theta_0\hat{x} + \sqrt{1-f}\,v_0\sin\theta_0\hat{y} = \vec{v}$ @ $y = fh$

$v = \sqrt{v_x^2 + v_y^2} = v_0\sqrt{\cos^2\theta_0 + (1-f)\sin^2\theta_0}$ (NOTE: v_0 when $f = 0$)

b) if $f = \frac{1}{2}$ $v = 28.0\sqrt{\cos^2 40 + \sin^2 40/2} = 24.9\ \frac{m}{s} = (21.4 + 3.5)\ \frac{m}{s}$

c) $(3.5/21.4)100 = 16.4\%$ more speed or $\frac{24.9}{21.4}100 = 116.4\%$ faster