STAEDTLER® No. 937 811E Engineer's Computation Pad

•••68 A cat rides a merry-go-round turning with uniform circular motion. At time $t_1 = 2.00$ s, the cat's velocity is $\vec{v}_1 = (3.00 \text{ m/s})\hat{i} + (4.00 \text{ m/s})\hat{j}$, measured on a horizontal xy coordinate system. At $t_2 = 5.00$ s, its velocity is $\vec{v}_2 = (-3.00 \text{ m/s})\hat{i} + (-4.00 \text{ m/s})\hat{j}$. What are (a) the magnitude of the cat's centripetal acceleration and (b) the cat's average acceleration during the time interval $t_2 - t_1$, which is less than one period?

NOTE: $\vec{v}_2 = -\vec{v}_1$

$\Rightarrow$ THEY OCCUR ON OPPOSITE SIDES OF A DIAMETER

$\Rightarrow t_2 - t_1$ IS HALF THE TIME TO GO AROUND

$t_2 - t_1 = T/2 =$ HALF THE PERIOD $= 5-2 = 3.00$ s

THE CONSTANT SPEED OF THE "UNIFORM CIRCULAR MOTION IS RELATED TO THE PERIOD & CIRCUMFERENCE BY: $v = \frac{\text{DISTANCE}}{\text{TIME}} = \frac{2\pi R}{T}$

$v = \|\vec{v}\| = \sqrt{v_x^2 + v_y^2} = \sqrt{(\pm 3)^2 + (\pm 4)^2} = \sqrt{5^2} = 5.00 \frac{m}{s}$

$\Rightarrow v = 5 = \frac{2\pi R}{T = 6.00 s} \Rightarrow R = \frac{vT}{2\pi} = \frac{5(6)}{2\pi} = 4.77$ m

NOTE: $\vec{v}$ IS TANGENT TO THE CIRCLE & THEREFORE PERPENDICULAR TO A RADIUS DRAWN TO IT

$\Rightarrow \vec{r} \cdot \vec{v} = 0 \Rightarrow (x\hat{x} + y\hat{y} + z\hat{z}) \cdot (v_x\hat{x} + v_y\hat{y} + v_z\hat{z})$

$= x v_x + y v_x + 0 = 0 \Rightarrow \frac{y}{x} = -\frac{v_x}{v_y} \left(= -\left(\frac{-3}{-4}\right) = -\left(\frac{+3}{+4}\right) = \frac{-3}{4} \right)$

$\Rightarrow$ slopes of $\vec{r}$ & $\vec{v}$ ARE NEGATIVE RECIPROCALS

a_c = CENTRIPETAL ACCELERATION $= v^2/R = \left(\frac{2\pi R}{T}\right)^2 / R = 4\pi^2 R / T^2$

$= 5^2/4.77 = 5.24 \text{ m/s}^2$

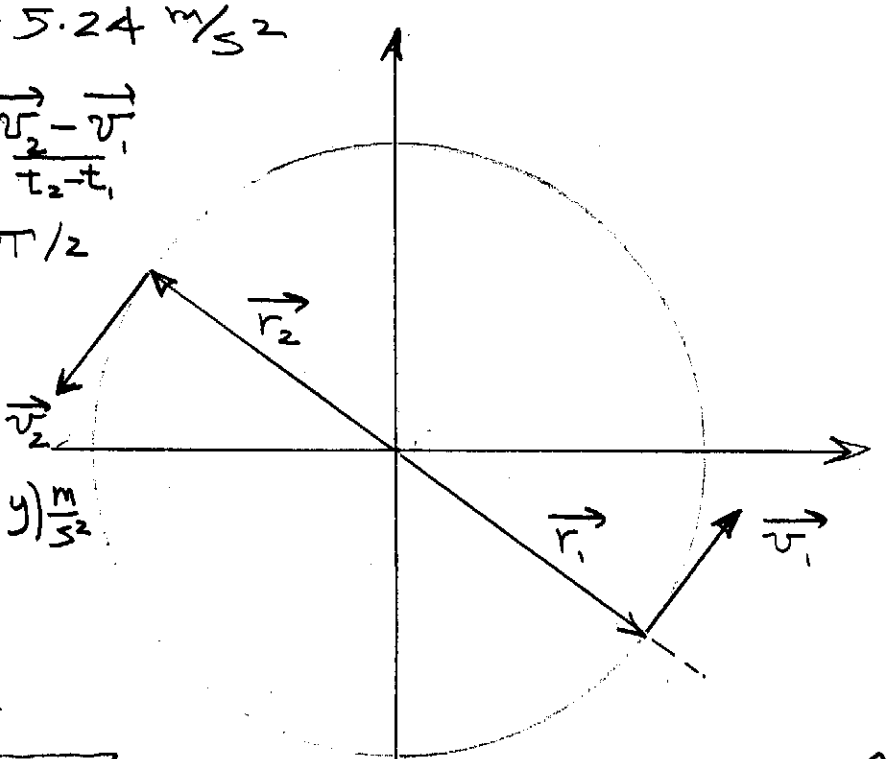
$\vec{a}_{AVG} \equiv \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$

$= \left[\vec{v}_2 - (-\vec{v}_2)\right] / T/2$

$= 4\vec{v}_2 / T$

$= \frac{2}{3}\left(\begin{matrix} -3.00\hat{x} \\ -4.00\hat{y} \end{matrix}\right)$

$= \left(-2.00\hat{x} + -2.67\,y\right) \frac{m}{s^2}$



$a_{AVG} = \sqrt{a_x^{AVG^2} + a_y^{AVG^2}} = 3.33 \text{ m/s}^2$ in the direction of $\hat{v}_2$ or $-\hat{v}_1$