STAEDTLER® No. 937 811E Engineer's Computation Pad

•••82 A 200-m-wide river has a uniform flow speed of 1.1 m/s through a jungle and toward the east. An explorer wishes to leave a small clearing on the south bank and cross the river in a powerboat that moves at a constant speed of 4.0 m/s with respect to the water. There is a clearing on the north bank 82 m upstream from a point directly opposite the clearing on the south bank. (a) In what direction must the boat be pointed in order to travel in a straight line and land in the clearing on the north bank? (b) How long will the boat take to cross the river and land in the clearing?

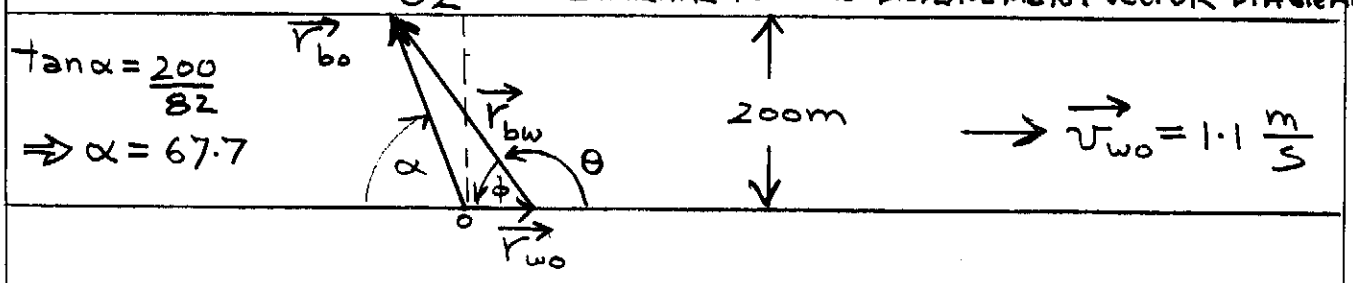
$\vec{v}_{bo} = \vec{v}_{bw} + \vec{v}_{wo}$ b=boat o=origin on land w=water

$(\vec{v}_{bo} = \vec{v}_{bw} + \vec{v}_{wo})(\Delta t = t)$

$= \vec{r}_{bo} = \vec{r}_{bw} + \vec{r}_{wo}$ DISPLACEMENT VECTORS FOR CONSTANT VELOCITY MOTION

$\vec{r}_{bo} = \vec{v}_{bw}\,t + \vec{v}_{wo}\,t$

SINCE $\vec{v}(\Delta t) = \frac{\Delta \vec{r}}{\Delta t}\Delta t = \Delta \vec{r}$ (SAME Δt for each) THE VELOCITY VECTOR DIAGRAM IS SIMILIAR TO THE DISPLACEMENT VECTOR DIAGRAM.



$\tan\alpha = \frac{200}{82}$

$\Rightarrow \alpha = 67.7$

$\vec{r}_{bo} = -82\hat{x} + 200\hat{y} = \vec{v}_{bw}\,t + \vec{v}_{wo}\,t$

$= [v_b\cos\theta\,\hat{x} + v_b\sin\theta\,\hat{y} + v_w\hat{x}]\,t$

$= [(-v_b\cos\phi + v_w)\hat{x} + v_b\sin\phi\,\hat{y}]\,t$

$\Rightarrow -82 = (-v_b\cos\phi + v_w)\,t$ & $200 = v_b\sin\phi\,t$

$\Rightarrow \sin\phi = 200/v_b t = (200/4)/t = 50/t$

$\Rightarrow \cos\phi = \sqrt{1-\sin^2\phi} = \sqrt{1 - 2500/t^2}$

$\therefore -82 = (-4\sqrt{1-2500/t^2} + 1.1)\,t$

$\Rightarrow 4\sqrt{1-2500/t^2} = 82/t + 1.1$

$[16(1 - 2500/t^2)]\,t^2 = [82^2/t^2 + 2(1.1)(82/t) + (1.1)^2]\,t^2$

$16t^2 - 40000 = 6724 + 180t + 1.21t^2$

$\Rightarrow 14.8t^2 - 180t - 46724 = 0 \Rightarrow t^2 - 12.2t - 3160 = 0$

$t = [12.2 \pm \sqrt{(12.2)^2 - 4(1)(-3160)}]/2 = [12.2 \pm 113]/2$

b) $t = 62.6$ sec to cross the River

$\sin\phi = 50/t = 50/62.6 = 0.798 \Rightarrow \phi = 53°$

a) $\Rightarrow \theta = 180 - 53 = 127°$ = This is the angle to point boat

$\vec{r}_{wo} = v_w\,t\,\hat{x} = 1.1(62.6) = 69\text{ m}$

$r_{bw} = \|\vec{r}_{bw}\| = v_b t = 4(62.6) = 250\text{ m} \overset{\text{check}}{=} \frac{200}{\sin\phi} = 250\text{m}\checkmark$