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••52 In designing circular rides for amusement parks, mechanical engineers must consider how small variations in certain parameters can alter the net force on a passenger. Consider a passenger of mass m riding around a horizontal circle of radius r at speed v. What is the variation dF in the net force magnitude for (a) a variation dr in the radius with v held constant, (b) a variation dv in the speed with r held constant, and (c) a variation dT in the period with r held constant?

FOR AN OBJECT OF MASS m, MOVING IN AN HORIZONTAL CIRCLE OF RADIUS r AT CONSTANT SPEED v, NEWTON'S 2ND LAW FOR THE RADIAL MOTION IS:

$$F_{radial}^{net} = F_c = m a_{c=radial} = m\frac{v^2}{r}$$

$F_c = F_c(v, r) \Rightarrow$ DIFFERENTIATE WRT r KEEPING v CONSTANT

a) $\frac{dF_c}{dr} = mv^2(-1)r^{-1-1} = -\frac{mv^2}{r^2} \Rightarrow dF_c = -\frac{mv^2}{r^2}dr$

$\Rightarrow$ THE CHANGE IN $F = dF \propto \frac{-dr}{r^2}$ (while holding v = constant)

IF $dr > 0$ ie. $r_f - r_i > 0$ then $dF < 0$ ie. F_c decreases

IF $dr < 0$ then $dF = F_f - F_i > 0$ ie. F_c increases

b) $\frac{dF_c}{dv} = \frac{2mv^{2-1}}{r} =$ DIFFERENTIATING F WHILE HOLDING (r = CONSTANT)

$\Rightarrow dF = \frac{2mv}{r}dv \Rightarrow dF \propto v\,dv$ (while holding r constant)

IN THIS CASE: IF v IS LARGE THEN THE CHANGE IN F IS LARGE

IN THE PREVIOUS CASE: IF r IS LARGE, THEN THE CHANGE IN F IS SMALL

TO FIND $\frac{dF_c}{dT}$ holding r CONSTANT, WE MUST EXPRESS $F_c = F_c(r, T)$

$v = \frac{2\pi r}{T} \Rightarrow \frac{v^2}{r} = \left(\frac{2\pi r}{T}\right)^2 / r = \frac{4\pi^2 r}{T^2}$

$\frac{dF_c}{dT} = \frac{d}{dT}\left\{\frac{m4\pi^2 r}{T^2}\right\}$ HOLDING r = CONSTANT $= -2m(4\pi^2 r)T^{-2-1} = \frac{-m8\pi^2 r}{T^3}$

$\Rightarrow dF_c = \frac{-m8\pi^2 r}{T^3}dT \Rightarrow dF \propto \frac{dT}{T^3}$

WHICH IS VERY SMALL FOR LARGE T but LARGE FOR VERY SMALL T