STAEDTLER® No. 937 811E Engineer's Computation Pad

•••33 The block in Fig. 7-11*a* lies on a horizontal frictionless surface, and the spring constant is 50 N/m. Initially, the spring is at its relaxed length and the block is stationary at position $x = 0$. Then an applied force with a constant magnitude of 3.0 N pulls the block in the positive direction of the x axis, stretching the spring until the block stops. When that stopping point is reached, what are (a) the position of the block, (b) the work that has been done on the block by the applied force, and (c) the work that has been done on the block by the spring force? During the block's displacement, what are (d) the block's position when its kinetic energy is maximum and (e) the value of that maximum kinetic energy?

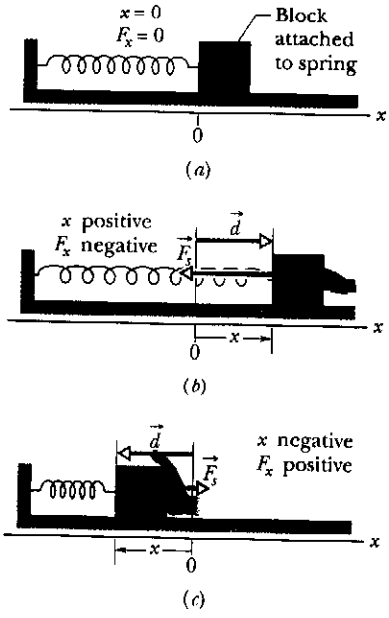


FIG. 7-11 (*a*) A spring in its relaxed state. The origin of an x axis has been placed at the end of the spring that is attached to a block. (*b*) The block is displaced by $\vec{d}$, and the spring is stretched by a positive amount x. Note the restoring force $\vec{F}_s$ exerted by the spring. (*c*) The spring is compressed by a negative amount x. Again, note the restoring force.

$$\vec{F}^{net}_{on\ block} = \vec{F}_{block,spring} + \vec{F}_{block,Applied} + m\vec{g} + \vec{N}_{block,surface}$$

EQUILIBRIUM IN VERTICAL DIRECTION

$$\Rightarrow m\vec{g} + \vec{N} = 0$$

$$W^{net}_{on\ block} = W_{b,s} + W_{b,A}$$

$$W_{b,s} = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

$$W_{b,A} = \vec{F}^{\,its\ constant}_{Applied} \cdot \vec{d}\ ;\ \vec{d} = (x_f - x_i)\hat{x}$$

$$\vec{F}_A = 3.0\,N\,\hat{x} \Rightarrow W_{b,A} = 3.0(x_f - x_i)$$

WORK ENERGY THEOREM: $W^{net}_{on\ m} = \Delta K$

$v_i \equiv 0$ & $v_f \equiv 0$ IN PARTS (a) & (b) & $x_i \equiv 0$

$$\Rightarrow W^{net} = 0 - \frac{1}{2}kx_f^2 + 3x_f = \Delta K \equiv 0$$

$$\Rightarrow x_f = 2(F_A = 3)/k = 6/50 = 0.12\ m \quad (a)$$

$$W_{b,A} = \vec{F}_{b,A} \cdot \vec{d} = F_A d = F_A x_f = (3.0)(0.12) = 0.36\ (Nm = Joules) \quad (b)$$

Since $W^{net}_{block} \equiv 0 \Rightarrow W_{b,s} = -W_{b,A}$ (c)

Check: $W_{b,s} = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2 = 0 - \frac{1}{2}50(0.12)^2$ ✓

WE HAVE FOR AN ARBITRARY POSITION

$$W^{net}_{block} = \Delta K \Rightarrow W_{b,s} + W_{b,A} = \Delta K$$

$$\Rightarrow \left(\frac{1}{2}kx_i^2 - \frac{1}{2}kx^2\right) + F_A x = \frac{1}{2}mv^2 - \frac{1}{2}mv_i^2\ ;\ x_i = v_i \equiv 0$$

$$K = \frac{1}{2}mv^2 = F_A x - \frac{1}{2}kx^2$$

K IS A MAXIMUM WHEN $\frac{dK}{dx} = 0$

$$\frac{dK}{dx} = F_A - \frac{1}{2}(2)kx = F_A - kx = 0 \Rightarrow x = F_A/k = \frac{3.0}{50} = 0.06\ m$$

NOTE $F^{net}_x = F_A + F_s = F_A + -kx = 0 \Rightarrow$ EQUILIBRIUM x value (d)

$$K = K_{max} = \frac{1}{2}mv^2_{max} = F_A x_{EQUIL} - \frac{1}{2}kx^2_{EQUIL}$$

$$= 3.0(0.06) - \frac{1}{2}(50)(0.06)^2 = 0.090\ (Nm = JOULES) \quad (e)$$