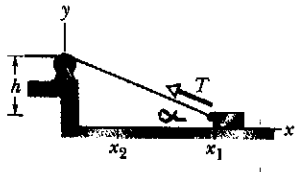
STAEDTLER® No. 937 811E Engineer's Computation Pad

•••42 Figure 7-42 shows a cord attached to a cart that can slide along a frictionless horizontal rail aligned along an $x$ axis. The left end of the cord is pulled over a pulley, of negligible mass and friction and at cord height $h = 1.20$ m, so the cart slides from $x_1 = 3.00$ m to $x_2 = 1.00$ m. During the move, the tension in the cord is a constant 25.0 N. What is the change in the kinetic energy of the cart during the move?




WORK-ENERGY THEOREM

$$W^{net}_{on\ cart} = \Delta K_{cart}$$

$$W^{net}_{on\ cart} \equiv \int_i^f \vec{F}_{net_{cart}} \cdot d\vec{r}$$

$$\vec{F}_{net_{cart}} = m\vec{g} + \vec{N} + \vec{T} \qquad d\vec{r} = dx\hat{x} + dy\hat{y} + dz\hat{z} = dx\hat{x} + 0\hat{y} + 0\hat{z}$$

$$W^{net}_{cart} = \int_i^f \{ m\vec{g}\cdot\hat{x} + \vec{N}\cdot\hat{x} + \vec{T}\cdot\hat{x} \}\, dx$$

$$= \int_{x_i}^{x_f} \{ 0 + 0 + T_x \}\, dx$$

$$T_x = -T\cos\alpha = -T \times \frac{x}{\sqrt{x^2+h^2}}$$

$$W^{net}_{cart} = \int_{x_i=3}^{x_f=1} -T\times\frac{x}{\sqrt{x^2+h^2}}\, dx$$

(YOU MIGHT FIND AN INTEGRAL IN THE BOOK'S APPENDIX = A-11)

let $u = x^2 + h^2$ then $\frac{du}{dx} = 2x \Rightarrow du = 2x\,dx$

$$\Rightarrow x\,dx = \frac{1}{2}du \quad \& \quad \frac{1}{\sqrt{x^2+h^2}} = \frac{1}{\sqrt{u}} = u^{-1/2}$$

If $x = 3 \Rightarrow u = x^2 + h^2 = 3^2 + 1.2^2 = 10.4$

$x = 1 \Rightarrow u = x^2 + h^2 = 1^2 + 1.2^2 = 2.44$

$$W^{net}_{cart} = \int_{u_i=10.4}^{u_f=2.44} -T\left(\frac{1}{2}du\right)\left(u^{-1/2}\right) = -\frac{T}{2}\int_{10.4}^{2.44} u^{-1/2}\, du$$

$$= -\frac{25}{2}\left\{\frac{u^{-1/2+1}}{-\frac{1}{2}+1}\right\}_{10.4}^{2.44} = -25\left[\sqrt{2.44} - \sqrt{10.4}\right]$$

$$W^{net}_{cart} = 41.6\ (\text{Nm} = \text{Joules})$$

$$\text{also } \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$