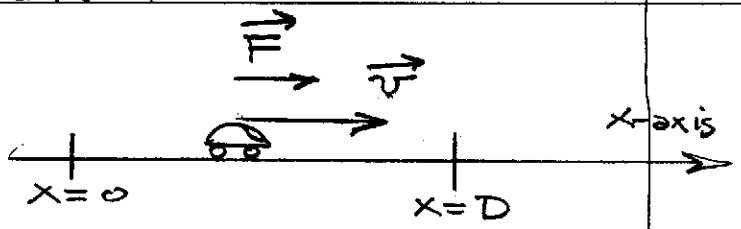
CONSTANT POWER MOTION

•••52 A funny car accelerates from rest through a measured track distance in time T with the engine operating at a constant power P. If the track crew can increase the engine power by a differential amount dP, what is the change in the time required for the run?



T = time to race the Distance = D

$$\text{POWER} = P = \vec{F}\cdot\vec{v} \quad \text{by Force}$$

$$= \frac{dK}{dt} \quad \text{if } \vec{F} = \vec{F}_{net}$$

$$P = \text{CONSTANT} \Rightarrow P = \frac{\Delta K}{\Delta t} = \frac{K_f - K_i}{T} = \frac{K_f = \frac{1}{2}mv_f^2}{T}$$

WE WANT $P = P(D,T)$ not $P = P(v_f, T)$

NOTE $v = \sqrt{\frac{2P}{m}}\,t^{1/2}$

So we can DIFFERENTIATE P WRT T HOLDING D CONSTANT

WE NEED A RELATIONSHIP $D = D(v_f, P)$, NOTE ...

$$D = \int_{x=0}^{x=D} dx = \int_0^{t=T} \frac{dx}{dt}\,dt = \int_0^T v\,dt$$

use $P = \vec{F}\cdot\vec{v} = Fv = mav = m\frac{dv}{dt}v$

$$\Rightarrow dt = \frac{mv}{P}dv$$

NOTE: $\boxed{a = P/(mv)}$

$$\Rightarrow D = \int_0^{v=v_f} \frac{mv^2}{P}\,dv = \frac{m}{P}\frac{v^3}{3}\Big|_0^{v_f} = \frac{1}{3}\frac{m}{P}v_f^3$$

$$\Rightarrow v_f = \left(\frac{3DP}{m}\right)^{1/3}$$

WE HAD $P = \frac{\frac{1}{2}mv_f^2}{T} = \frac{\frac{1}{2}m\left(\frac{3DP}{m}\right)^{2/3}}{T}$

$$\Rightarrow (2PT)^3 = m^3\left(\frac{3DP}{m}\right)^2$$

$$\Rightarrow P = \frac{9}{8}m\frac{D^2}{T^3}$$

NOTE: $D = x = \sqrt{\frac{8P}{9m}}\,t^{3/2=1.5}$

$$\frac{dP}{dT} = -3\,\frac{9}{8}m\frac{D^2}{T^4} = -\frac{3P}{T}$$

$$dP = -\frac{3P}{T}dT \Rightarrow dT = -\frac{T}{3P}dP$$

$$dT = -\frac{1}{3}T\frac{dP}{P}$$

STAEDTLER® No. 937 811E Engineer's Computation Pad