STAEDTLER® No. 937 811E Engineer's Computation Pad

•••36 A boy is initially seated on the top of a hemispherical ice mound of radius $R = 13.8$ m. He begins to slide down the ice, with a negligible initial speed (Fig. 8-49). Approximate the ice as being frictionless. At what height does the boy lose contact with the ice?

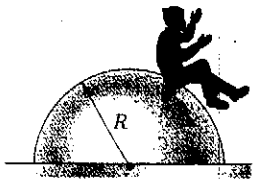
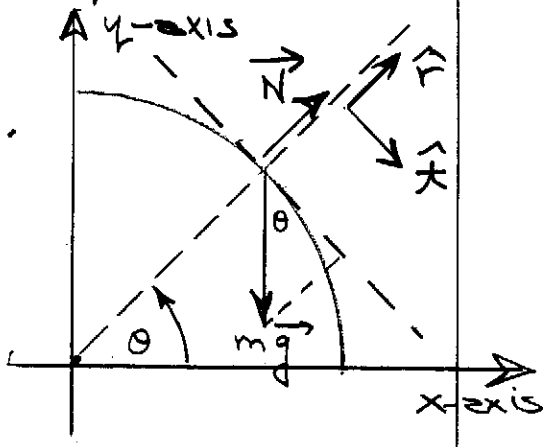


FIG. 8-49 Problem 36.

$\vec{F}_{net\ on\ boy} = m_{boy}\vec{a}_{boy}$

$\vec{F}_{net} = m\vec{g} + \vec{N}$



THE NORMAL FORCE IS RADIAL.

THE TANGENTIAL FORCE IS ONLY DUE TO GRAVITY:

$F^{net}_{tan} = mg\cos\theta = ma_{tan}$

THE TANGENTIAL ACCELERATION IS DUE TO A CHANGING SPEED

$a_{tan} = \frac{dv}{dt}$ BECAUSE $\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv\hat{t}}{dt}$ where $\hat{t}$ = unit TANGENT VECTOR

$\vec{a} = \frac{dv}{dt}\hat{t} + v\frac{d\hat{t}}{dt} = a_{tan}\hat{t} + v\frac{v}{R}(-\hat{r})$ where $\hat{r} = \hat{n}$ = UNIT NORMAL VECTOR

$\vec{a} = a_{tan}\hat{t} + a_{centripetal}$ ($-\hat{r}$ = points to Center)

THE NET RADIAL FORCE $= F_c = ma_c = m\frac{v^2}{R}$

$F_c^{net} = mg\sin\theta - N = m\frac{v^2}{R}$

$\Rightarrow N = mg\sin\theta - m\frac{v^2}{R}$ = FORCE OF ICE ON BOY

THE BOY LOOSES CONTACT WITH ICE WHEN $N = 0$

$\Rightarrow mg\sin\theta - m\frac{v^2}{R} = 0 \Rightarrow \frac{1}{2}mv^2 = \frac{1}{2}mgR\sin\theta$ *

CONSERVATION OF MECHANICAL ENERGY: $E_i = E_f$

$K_i + U_i = K_f + U_f$ APPLY WHERE i = @ top & f = CONTACT LOSS

$0 + mgR = \frac{1}{2}mv^2 + mgy$ ($y_i = R$, $v_i = 0$ & $y_f = y$, $v_f = v$)

$\Rightarrow \frac{1}{2}mv^2 = mg(R - y) = mg(R - R\sin\theta)$

$= mgR(1 - \sin\theta)$

Eqn * ALSO $= \frac{1}{2}mgR\sin\theta$ (FROM NEWTON'S LAWS)

$\Rightarrow 2(1 - \sin\theta) = \sin\theta \Rightarrow 2 = 3\sin\theta \Rightarrow \sin\theta = \frac{2}{3} \Rightarrow \theta = 41.8°$

$\Rightarrow h = R\sin\theta = 2R/3 = 9.2$ m