STAEDTLER® No. 937 811E Engineer's Computation Pad

•••65 The cable of the 1800 kg elevator cab in Fig. 8-59 snaps when the cab is at rest at the first floor, where the cab bottom is a distance $d = 3.7$ m above a spring of spring constant $k = 0.15$ MN/m. A safety device clamps the cab against guide rails so that a constant frictional force of 4.4 kN opposes the cab's motion. (a) Find the speed of the cab just before it hits the spring. (b) Find the maximum distance x that the spring is compressed (the frictional force still acts during this compression). (c) Find the distance that the cab will bounce back up the shaft. (d) Using conservation of energy, find the approximate total distance that the cab will move before coming to rest. (Assume that the frictional force on the cab is negligible when the cab is stationary.)

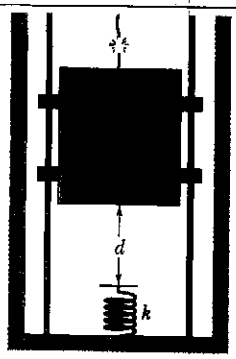


FIG. 8-59
Problem 65.

$W^{net}_{CAB} = \Delta K_{CAB}$

$W^{net}_{CAB} = W_G + W_S + W_f$

$= -\Delta U_G - \Delta U_S + W_f$

$\Rightarrow W_f = \Delta K + \Delta U_G + \Delta U_S$

$W_f = \Delta E_{mec}$

$E_{mech} = K + U_S + U_G$

$\Delta E_{thermal} = -W_f$

$\Rightarrow \Delta E_{mech} + \Delta E_{thermal} = \Delta E_{total} = 0$

$W_f = \Delta E_{mec} \Rightarrow \vec{f}\cdot\vec{d} = \Delta K + \Delta U_G + (\Delta U_S = 0 \text{ in part a})$

$\Rightarrow -fd = \left(\frac{1}{2}mv_f^2 - 0\right) + \left[\Delta(mgy) = mg\Delta y = mg(-d)\right]$

a) $v_f = \sqrt{\frac{2}{m}\left[mgd - fd\right]} = \sqrt{\frac{2}{1800}\left[1800(9.80)(3.7) - 4400(3.7)\right]} = 7.4\ \frac{m}{s}$

$W_f = \Delta E_{mec} = \Delta K + \Delta U_G + \Delta U_S$ use i = initial = @ spring & f = final = max compression

$\Rightarrow -fx = \left(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2\right) + \left[\Delta(mgy) = mg\Delta y = -mgx\right] + \left[\frac{1}{2}kx^2 - 0\right]$

$\frac{1}{2}kx^2 + (f - mg)x - \frac{1}{2}mv_i^2 = 0$ NOTE $v_f = 0$ (stops) $\left.\begin{matrix} x_i = 0 \\ x_f = x \end{matrix}\right\}$ SPRING

$x^2 + \frac{2(f-mg)}{k}x - \frac{m}{k}v_i^2 = 0 \Rightarrow x = \frac{1}{2}\left[-\frac{2(f-mg)}{k} \pm \sqrt{\left(\frac{2(f-mg)}{k}\right)^2 + 4\frac{m}{k}v_i^2}\right]$

b) $x = \frac{1}{2}\left[+0.177 \pm \sqrt{0.0317 + 0.048(v_i = 7.4)^2}\right] = \frac{0.177 \pm 1.63}{2} = 0.90\text{ m}$

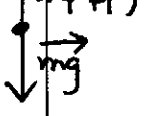
$W_f = \Delta K + \Delta U_G + \Delta U_S$ with i = max comp of spring & Δy = max height

$-f\Delta y = \left[\frac{1}{2}m(v_f = 0)^2 - \frac{1}{2}m(v_i = 0)^2\right] + \left[\Delta(mgy) = mg\Delta y\right] + \left[\frac{1}{2}k(x = 0)^2 - \frac{1}{2}kx^2\right]$

$\Rightarrow \Delta y = y_f - y_i = \left(\frac{1}{2}kx^2\right)/(mg + f) \overset{x = 0.90\text{m}}{=\!=\!=} 2.76 = 2.8\text{ m}$

WHEN MOTION STOPS, EQUILIBRIUM VERTICALLY $\Rightarrow mg = k|x_f|$

NET VERTICAL DISPLACEMENT FROM BEGINNING TO END $= \Delta y = -(d + |x_f|)$



$W_f = -fD_{total} = \left[\Delta K = 0 - 0 = 0\right] + \left[\Delta U_G = mg\Delta y\right] + \left[\Delta U_S = \frac{1}{2}kx_f^2 - 0\right]$

$D = \left[mg(-\Delta y = d + |x_f| = d + \frac{mg}{k}) - \frac{1}{2}k(x_f = -\frac{mg}{k})^2\right]/f$

$= \left[6.73\times10^4 - 1.04\times10^3\right]/4400 = 15\text{ m}$