STAEDTLER® No. 937 811E Engineer's Computation Pad

•••69 A small ball of mass m is aligned above a larger ball of mass $M = 0.63$ kg (with a slight separation, as with the baseball and basketball of Fig. 9-70*a*), and the two are dropped simultaneously from a height of $h = 1.8$ m. (Assume the radius of each ball is negligible relative to h.) (a) If the larger ball rebounds elastically from the floor and then the small ball rebounds elastically from the larger ball, what value of m results in the larger ball stopping when it collides with the small ball? (b) What height does the small ball then reach (Fig. 9-70*b*)?

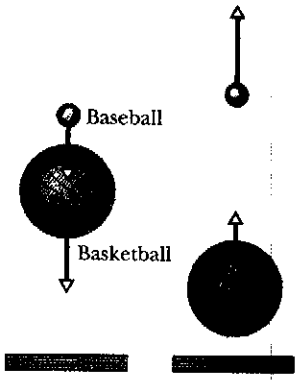


FIG. 9-70 Problem 69.

IF THE LARGE BALL REBOUNDS ELASTICALLY FROM THE FLOOR (WHICH DOES NOT MOVE), THEN IT REBOUNDS WITH THE SAME SPEED IT IMPACTS IT WITH:

$$v^2 = v_0^2 - 2g(y - y_0)$$

$$\Rightarrow y_0 - y \equiv h = \frac{v_{IMPACT}^2}{2g}$$

THIS COULD ALSO HAVE BEEN ARRIVED AT WITH CONSERVATION OF ENERGY $Mgh = \frac{1}{2}Mv_I^2$ ALONG WITH THE RESULTING COLLISION OF M WITH THE EARTH & SINCE $M_{EARTH} \gg M \Rightarrow v_{M\,final} = v_I = \sqrt{2gh}$ IN REVERSED DIRECTION

ASSUMING THAT THE BALL'S RADII ARE r & $R \ll h$ AND THE INITIAL SEPARATION IS SMALL — THAT SMALL SEPARATION IS MAINTAINED SINCE BOTH FALL IDENTICALLY AT SPEEDS INDEPENDENT OF THEIR MASSES THEN THE ELASTIC COLLISION BETWEEN THE BALLS IS WITH OPPOSITE VELOCITIES OF THE SAME MAGNITUDE i.e. $v = \sqrt{2gh}$

$$\vec{P}_{initial}^{\,total} = \vec{P}_{final}^{\,total} \Rightarrow v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i} \;\&\; v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i}$$

ASIDE: $v_{2f} - v_{1f} = \frac{m_1 + m_2}{m_1 + m_2} v_{1i} + \frac{-(m_1 + m_2)}{m_1 + m_2} v_{2i} = v_{1i} - v_{2i}$

$\Rightarrow$ RELATIVE VELOCITY OF APPROACH $= v_{1i} - v_{2i} = -(v_{1f} - v_{2f})$
= NEGATIVE OF RELATIVE VELOCITY OF RECESSION

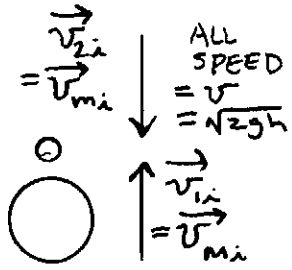
a) $v_{Mf} = \frac{M - m}{m + M}\sqrt{2gh} + \frac{2m}{m + M}\left(-\sqrt{2gh}\right) = \frac{M - 3m}{m + M}\sqrt{2gh} = 0$ if $3m = M$

$v_{mf} = \frac{2M}{m + M}\sqrt{2gh} + \frac{m - M}{m + M}\left(-\sqrt{2gh}\right) = \frac{3M - m}{m + M}\sqrt{2gh} \overset{3m = M}{=} \frac{8m}{4m}\sqrt{2gh}$

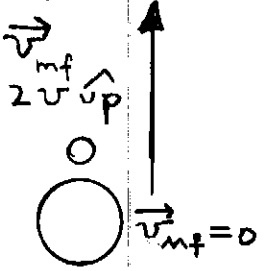
$v_{mf} = 2\sqrt{2gh}$; $v^2 = v_0^2 - 2g(y - y_0) \Rightarrow H = (y - y_0) = v_0^2/2g = \frac{8gh}{2g} = 4h$

OR $E_i = E_f : K_M^i + U_M^i + K_m^i + U_m^i = K_M^f + U_M^f + K_m^f + U_m^f$

b) $\Rightarrow 0 + 0 + \frac{1}{2}m\left(v = 2\sqrt{2gh}\right)^2 + 0 = 0 + 0 + 0 + mgH \Rightarrow H = 4h$ ✓



BEFORE BALLS COLLIDE



AFTER BALLS COLLIDE

NOTE THE RELATIVE VELOCITY OF APPROACH $= v_{1i} - v_{2i} = 2v\,\hat{up}$
$=$ "RELATIVE VELOCITY OF SEPARATION" $= -(v_{1f} - v_{2f}) = 2v\,\hat{up}$