STAEDTLER® No. 937 811E Engineer's Computation Pad

•78 A 6090 kg space probe moving nose-first toward Jupiter at 105 m/s relative to the Sun fires its rocket engine, ejecting 80.0 kg of exhaust at a speed of 253 m/s relative to the space probe. What is the final velocity of the probe?

$$\frac{d\vec{p}}{dt} = \frac{d(M\vec{v})}{dt} = \vec{F}_{\text{on rocket}}^{\text{net}}$$

$$\frac{dM\vec{v}}{dt} = \vec{U}\frac{dM}{dt} + \vec{F}_{ext} \qquad \vec{F}_{ext} = \vec{F}_{\text{gravitation}} \overset{\text{set}}{=} 0$$

maybe not a good assumption!

$$\vec{U} = \vec{v}_{rel} + \vec{v}$$

$\vec{U}$ = exhaust products relative to Sun

$\vec{v}_{rel}$ = velocity of exhaust products relative to Rocket

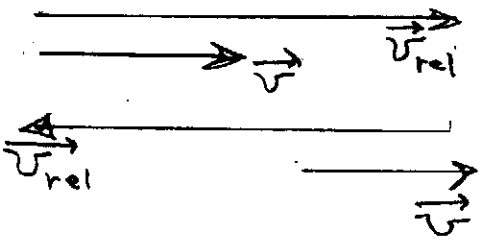
$\vec{v}$ = velocity of Rocket relative to Sun.

$$\Rightarrow \frac{dM}{dt}\vec{v} + M\frac{d\vec{v}}{dt} = \vec{v}_{rel}\frac{dM}{dt} + \vec{v}\frac{dM}{dt}$$

$$\Rightarrow M\frac{d\vec{v}}{dt} = \vec{v}_{rel}\frac{dM}{dt}$$

If the Rocket wants to slow

If the Rocket wants to speed up



TAKE $\vec{v}$ as positive direction

$$M\frac{dv}{dt} = \pm v_{rel}\frac{dM}{dt}$$ + for slower, − for faster.

THRUST $= v_{rel}\frac{dM}{dt}$; IN GENERAL IS NOT FORCE ON ROCKET $= \vec{U}\frac{dM}{dt}$

It is when $v = 0$ (i.e. @ take off)

$$dv = \pm v_{rel}\frac{dM}{M} \Rightarrow \int_{v_i}^{v_f} dv = \int_{M_i}^{M_f} v_{rel}\frac{dM}{M}$$

$$v_f - v_i = \pm v_{rel}\left[\ln M_f - \ln M_i = \ln(M_f/M_i)\right]$$

$$= \pm 253 \ln\left(\frac{6090-80}{6090}\right) = \pm 253(-0.0132)$$

$$= \mp 3.35 \frac{m}{s}$$

$$\Rightarrow v_f = 105 + 3 = 108 \text{ m/s}$$

or $105 - 3 = 102$ m/s