STAEDTLER® No. 937 811E Engineer's Computation Pad

•••8 A uniform soda can of mass 0.140 kg is 12.0 cm tall and filled with 1.31 kg of soda (Fig. 9-43). Then small holes are drilled in the top and bottom (with negligible loss of metal) to drain the soda. What is the height h of the com of the can and contents (a) initially and (b) after the can loses all the soda? (c) What happens to h as the soda drains out? (d) If x is the height of the remaining soda at any given instant, find x when the com reaches its lowest point.

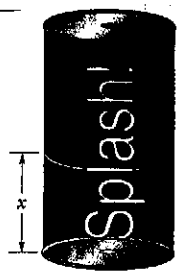
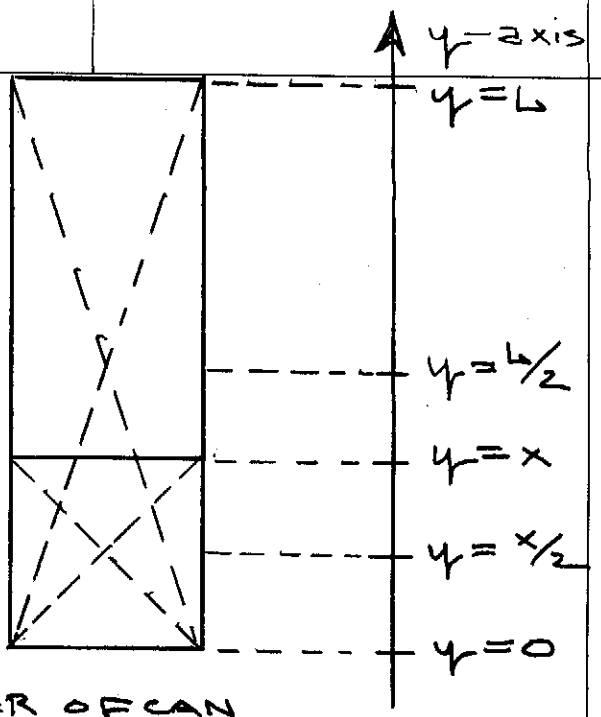


FIG. 9-43
Problem 8.



$$\vec{r}_{com} \equiv \sum_{i=1}^{n} m_i \vec{r}_i / M$$

$$\Rightarrow \vec{r}_{com} = \left(m_{can}\vec{r}^{\,com}_{can} + m_{liq}\vec{r}^{\,com}_{liquid}\right)/M$$

SYMMETRY $\Rightarrow \vec{r}^{\,com}_{can}$ @ CENTER OF CAN

$\Rightarrow r^{com}_{liquid}$ @ CENTER OF LIQUID DISTRIBUTION

THEREFORE: $y_{com} = \left(m_{can}\frac{L}{2} + m_{liq}\frac{x}{2}\right)/(m_{can}+m_{liq})$

$$m_{liquid} = \rho_{liq}V_{liq} = \left(\frac{1.31\,kg}{\pi r_{can}^2 L}\right)\left(\pi r_{can}^2 x\right) = \left(1.31\frac{x}{L}\right)kg = m_l\frac{x}{L}$$

Where $\rho_{liq} = \dfrac{\text{TOTAL MASS OF LIQUID}}{\text{TOTAL VOLUME OF CAN}}$

$$y_{com} = \left[m_c\frac{L}{2} + \left(m_l\frac{x}{L}\right)\frac{x}{2}\right] \Big/ \left(m_{can} + m_l\frac{x}{L}\right)$$

a) $y_{com}(x=L) = \left[\left(m_c\frac{L}{2} + m_l\frac{L}{2}\right)/(m_c+m_l)\right] = \frac{L}{2}$ ✓

b) $y_{com}(x=0) = \left[m_c L/2 + 0\right]/(m_{can}+0) = \frac{L}{2}$ ✓

FIND THE MINIMUM OF $y_{cm} \Rightarrow \left.\dfrac{dy_{com}}{dx}\right|_{@x=\text{sought after}} = 0$

$$\frac{dy_{com}}{dx} = \frac{\left(m_c + m_l x/L\right)\left(m_l/L\right) - \left(m_c L/2 + m_l x^2/2L\right)m_l/L}{\left(m_{can} + m_l x/L\right)^2}$$

$= 0$ when $m_c m_l \frac{x}{L} + m_l^2\frac{x^2}{L^2} - m_c m_l/2 - m_l^2\frac{x^2}{2L^2} = 0$

$$\frac{m_l^2}{2L^2}x^2 + \frac{m_c m_L}{L}x - \frac{m_c m_l}{2} = 0 \quad \text{or} \quad x^2 + \frac{2Lm_c}{m_l}x - \frac{L^2 m_c}{m_l} = 0$$

$$x = \frac{1}{2}\left[-\frac{2Lm_c}{m_l} \pm \sqrt{\frac{4L^2m_c^2}{m_l^2} + \frac{4L^2m_c}{m_l}}\right] = \frac{1}{2}\left[\frac{-2(.12)(.14)}{1.31} \pm \sqrt{(.0256)^2 + (0.0256)\left[2(.12)\right]}\right]$$

$x = 0.0284\,m = 2.8\,cm$